The University of Manchester

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Jet vetoing as a probe of new physics

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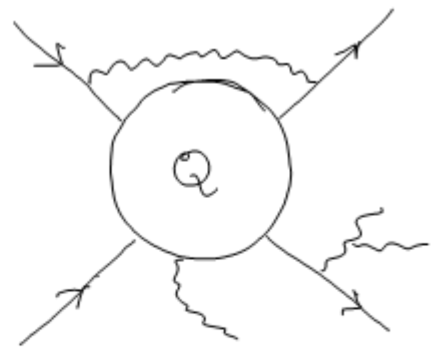
- An introduction to soft gluons
- Jet vetoing: (i) dijets; (ii) Higgs; (iii) TeV-scale resonances.
- [Superleading logarithms]

JF, A. Kyrieleis, M. Seymour: JHEP 0608:059, 2006.
JF, M. Sjödahl: JHEP 0709:119, 2007.
JF, A. Kyrieleis, M. Seymour: JHEP 0809:128, 2008.
JF, J. Keates, S. Marzani, JHEP 0907:023, 2009.
B. Cox, JF, A. Pilkington, Phys. Lett. B696 (2011) 87.
R. Delgado, JF, S. Marzani, M. Seymour, JHEP 1108 (2011) 157.
S. Ask, J.H. Collins, JF, A. Pilkington, arXiv:1108.2396 [hep-ph].

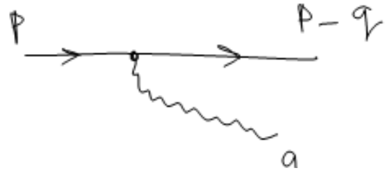
Given a particular hard scattering process we can ask how it will be dressed with additional radiation (perturbatively calculable):



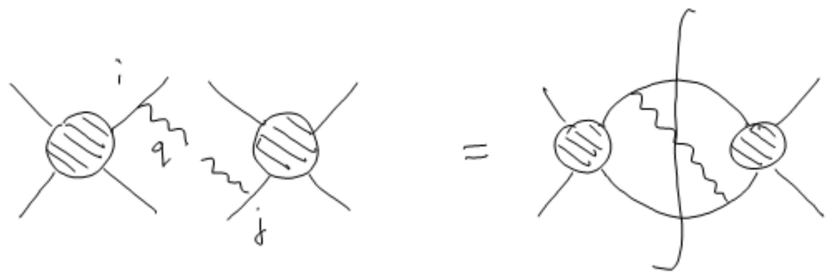
This question may not be interesting a priori because hadronization could wreck any underlying partonic correlations. However experiment reveals that the hadronization process is ‘gentle’.

The most important emissions are those involving either <u>collinear</u> quarks/gluons or <u>soft</u> gluons. By important we mean that the usual suppression in the strong coupling is compensated by a large logarithm.

SOFT GLUONS



$$= 2 g p_\mu \delta_{\lambda\lambda'} T^a_{ij}$$

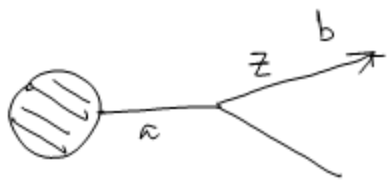


$$d\sigma_{n+1} = d\sigma_n \frac{\alpha_s}{2\pi} \frac{dE}{E} \frac{d\Omega}{2\pi} \sum_{ij} C_{ij} E^2 \, \frac{p_i \cdot p_j}{p_i \cdot q \; p_j \cdot q}$$

- Only have to consider soft gluons off the external legs of a hard subprocess since internal hard propagators cannot be put on shell.
- Virtual corrections are included analogously….of which more later….
- Only need to consider gluons.
- Colour factor is the “problem”.

COLLINEAR EMISSIONS

Colour structure is easier. It is as if emission is off the parton to which it is collinear ~ "classical branching".



$$d\sigma_{n+1} = d\sigma_n \, \frac{\alpha_s}{2\pi} \frac{dq^2}{q^2} dz \; P_{ba}(z)$$

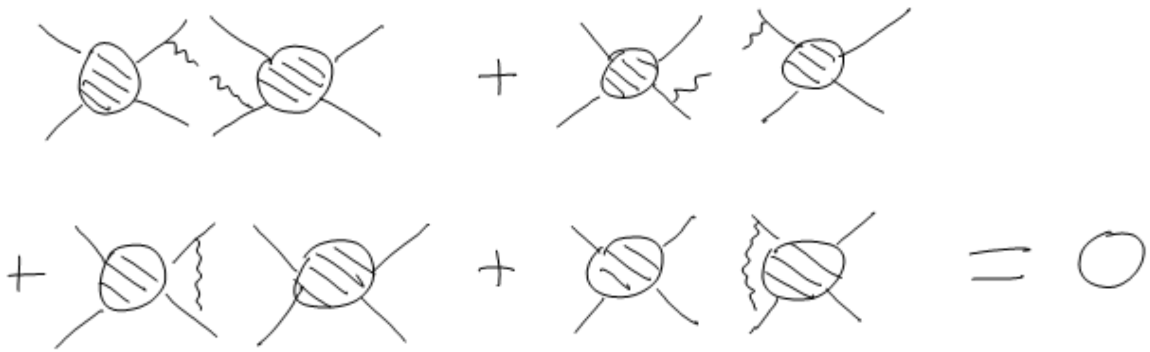
In the Monte Carlos: soft and/or collinear evolution is handled simultaneously using "angular ordered parton evolution".

Conventional wisdom: OK but only in the large N_c approximation where colour simplifies hugely.
Also assumes azimuthal averaging.

Not all observables are affected by soft and/or collinear enhancements

Intuitive: imagine the e^+e^- total cross-section. It cannot care that the outgoing quarks may subsequently radiate additional soft and/or collinear particles (causality and unitarity).

Bloch-Nordsieck: *soft gluon corrections cancel in "sufficiently inclusive" observables.*



Miscancellation can be induced by restricting the real emissions in some way.

All observables are "sufficiently inclusive" to guarantee that the would-be soft divergence cancels (no detector can detect zero energy particles). But the miscancellation may leave behind a logarithm, e.g. if real emissions are forbidden above μ then virtual corrections give

$$\alpha_s \int_\mu^Q \frac{dE}{E} = \alpha_s \ln \frac{Q}{\mu}$$

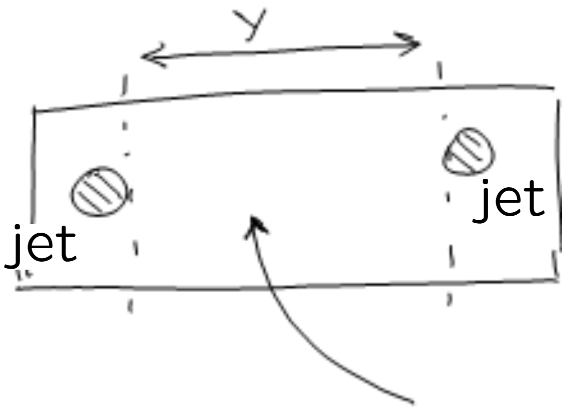
Soft gluon corrections will be important for observables that insist on only *small deviations from lowest order kinematics*.

In such cases real radiation is constrained to a small corner of phase space and BN miscancellation induces large logarithms.

If V measures 'distance' from the lowest order kinematics:

Event shapes such as thrust ($V = 1 - T$)
Production near threshold (top, W/Z) ($V = 1 - M^2/\hat{s}$)
Drell-Yan at low p_T (W/Z or Higgs) ($V = p_T^2/\hat{s}$)
Deep-inelastic scattering at large x ($V = 1 - x$)
Gaps between jets....

JET VETOING: "Gaps between jets"



Jets produced with $p_T = Q \gg Q_0$

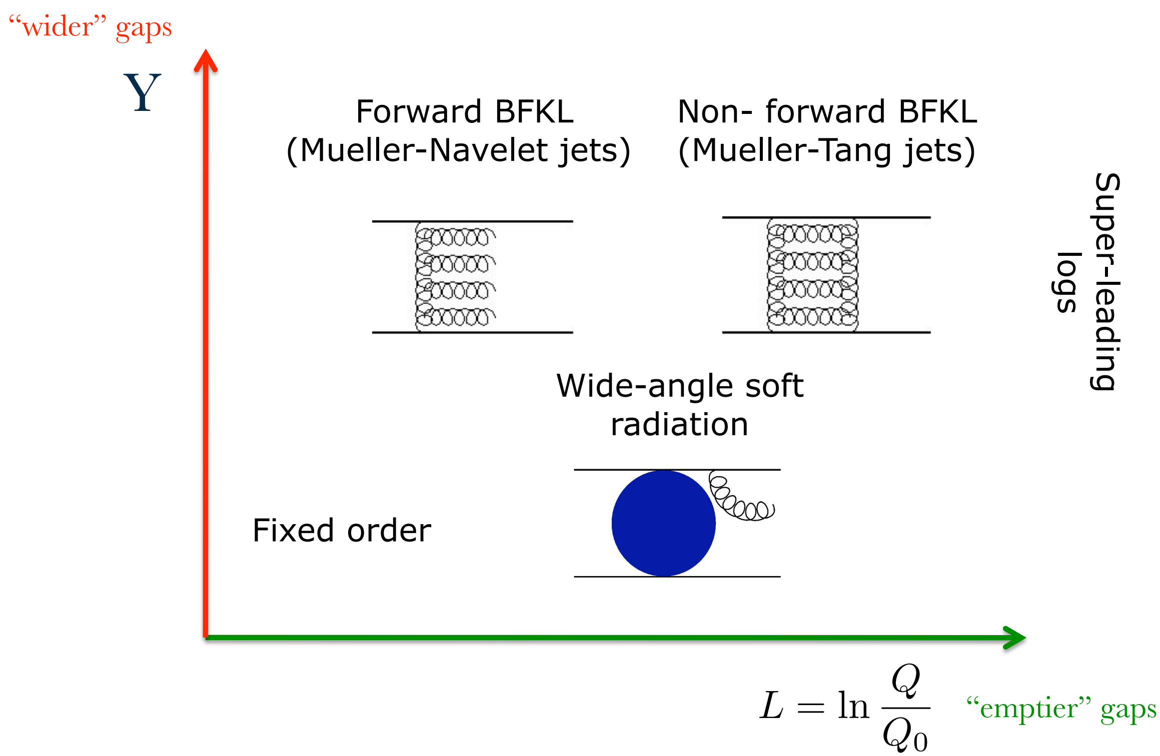
No radiation in between jets with $k_T > Q_0$

Observable restricts emission in the gap region therefore expect

$$\alpha_s^n \ln^n(Q/Q_0)$$

i.e. do not expect collinear enhancement since we sum inclusively over the collinear regions of the incoming and outgoing partons.

The rich physics of “gaps between jets”…..



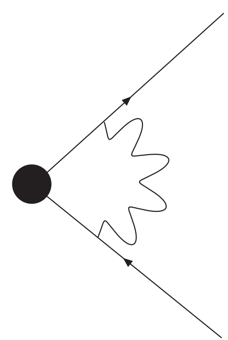
Real emissions are forbidden in the phase-space region

$$-Y/2 < y < Y/2$$
$$k_T > Q_0$$

"*By Bloch-Nordsieck, all other real emissions cancel and we therefore only need to compute the <u>virtual</u> soft gluon corrections to the primary hard scattering*."

$e^+e^- \to q\bar{q}$ case is very simple:

$$\sigma_{\text{gap}} = \sigma_0 \, \exp\left(-C_F \frac{\alpha_s}{\pi} Y \ln\left(\frac{Q}{Q_0}\right)\right)$$



The virtual gluon is integrated over "in gap" momenta, i.e. the region where real emissions are forbidden.

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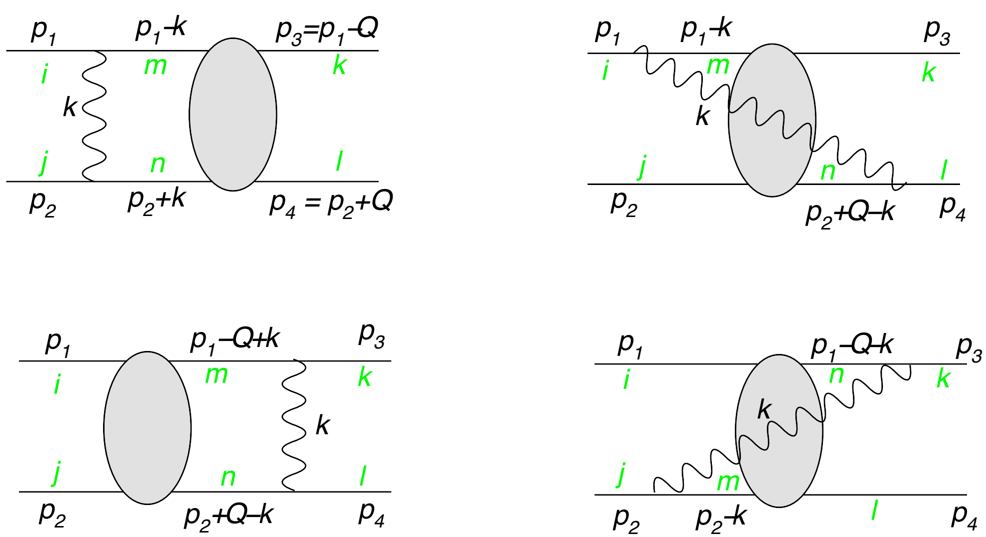


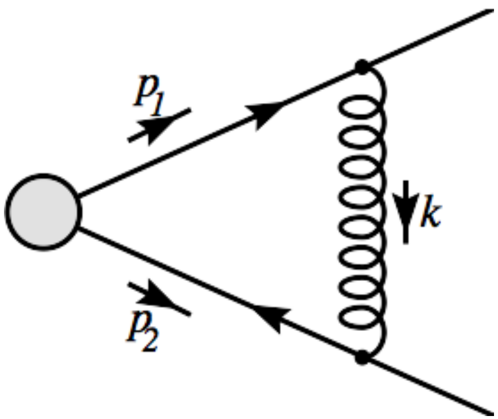
The virtual gluon is integrated over “in gap” momenta, i.e. the region where real emissions are forbidden.

(plus two others)

But this is too naïve….as we shall soon see

e^+e^- revisited

The colour structure is simple enough that the Coulomb gluons lead only to a phase both above & below Q_0.



$$\mathcal{A}_1 = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F\,(Y - i\pi)\,\mathcal{A}_0$$

eikonal $k^2=0$

Coulomb $p_1^{\,2}=p_2^{\,2}=0$

$$\mathcal{A} = e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y-i\pi)}\mathcal{A}_0$$

$$\sigma = \mathcal{A}^\star\mathcal{A} = \mathcal{A}_0^\star e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y+i\pi)}e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y-i\pi)}\mathcal{A}_0$$

i π terms cancel

Back to hadron-hadron collisions…

The amplitude can be projected onto a colour basis:

$$(M)_{ij}^{kl} = M^{(1)} C_{ijkl}^{(1)} + M^{(8)} C_{ijkl}^{(8)}$$

$$C_{ijkl}^{(8)} = (T^a)_{ik}(T^a)_{jl}$$
$$C_{ijkl}^{(1)} = \delta_{ik}\delta_{jl}.$$

i.e. $\mathbf{M} = \begin{pmatrix} M^{(1)} \\ M^{(8)} \end{pmatrix}$ and $\sigma = \mathbf{M}^\dagger \mathbf{S}_V \mathbf{M}$

$$\mathbf{S}_V = \begin{pmatrix} N^2 & 0 \\ 0 & \frac{N^2-1}{4} \end{pmatrix}$$

Iterating the insertion of soft virtual gluons builds up the N$^{\text{th}}$ order amplitude:

$$\mathbf{M} = \exp\left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{dk_T}{k_T} \, \mathbf{\Gamma} \right) \mathbf{M}_0$$

The factorial needed for exponentiation arises as a result of ordering the transverse momenta of successive soft gluons, i.e.

$$Q_0 \ll k_{T1} \ldots \ll k_{TN} \ll Q$$

where the evolution matrix is

$$\mathbf{\Gamma} = \begin{pmatrix} \frac{N^2-1}{4N}\rho(Y,\Delta y) & \frac{N^2-1}{4N^2} i\pi \\ i\pi & -\frac{1}{N} i\pi + \frac{N}{2} Y + \frac{N^2-1}{4N}\rho(Y,\Delta y) \end{pmatrix}$$

$\Delta y =$ distance between jet centres $\quad Y =$ size of gap

In qq → qq the colour structure is more complicated than e^+e^- and the Coulomb gluons no longer exponentiate into a phase above Q_0 (due to the presence of the real parts of the virtual corrections).

$$\mathbf{\Gamma} = \begin{pmatrix} \frac{N^2-1}{4N}\rho(Y,\Delta y) & \frac{N^2-1}{4N^2} i\pi \\ i\pi & -\frac{1}{N} i\pi + \frac{N}{2} Y + \frac{N^2-1}{4N}\rho(Y,\Delta y) \end{pmatrix}$$

Coulomb gluons are relevant

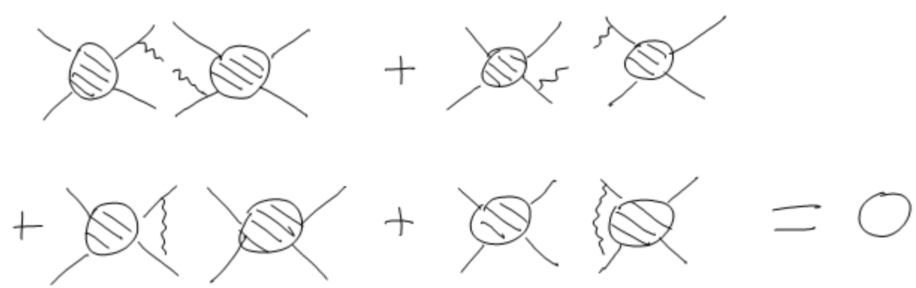
Coulomb gluons

a.k.a. Glauber gluons

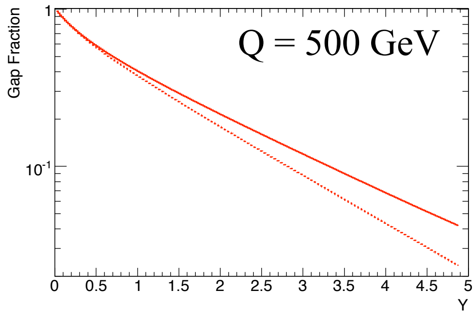
- We have skipped over a subtle issue…..the real-virtual cancellation of soft gluons occurs point-by-point in (y, k_T) only between the *real parts* of the virtual correction and the real emission.

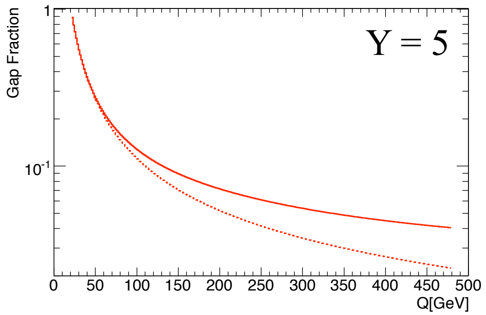
- The imaginary part obviously cancels if the soft gluon is closest to the cut….but what about subsequent evolution? Might this spoil the real-virtual cancellation below Q_0?

- No, it does not. The "non-cancelled" iπ terms exponentiate to produce a pure phase in the amplitude → no physical effect.

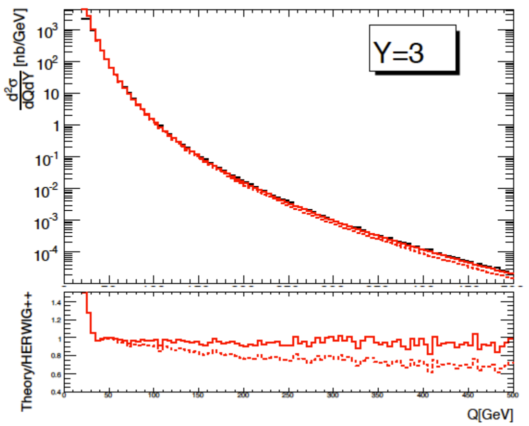


How well can we calculate?





Comparison to HERWIG++



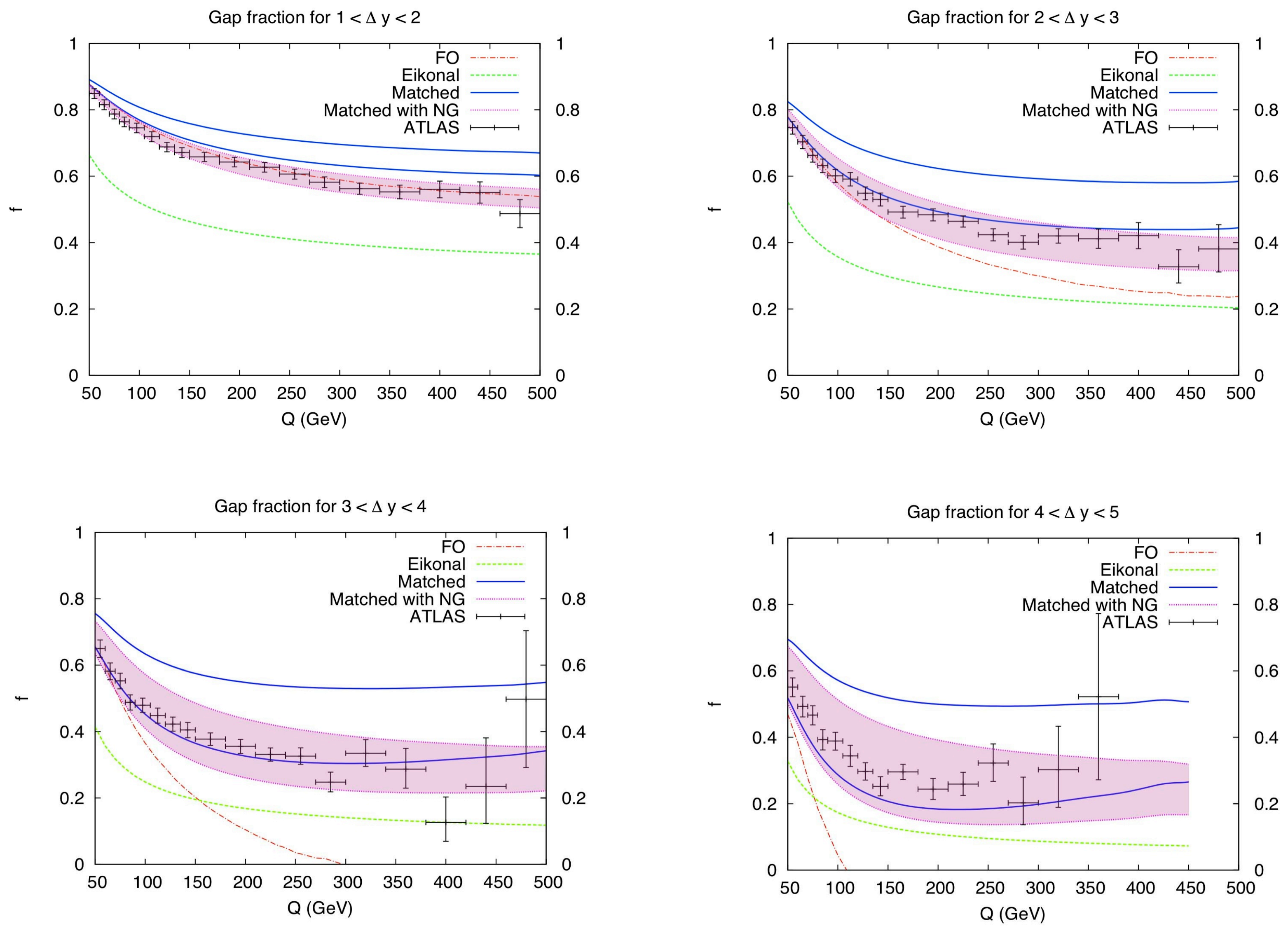
Solid = resummation of “primary” logs

Dotted = Dropping all Coulomb gluon contributions

All for LHC at 14 TeV

Agreement with Herwig is accidental
* Coulomb gluons
* Energy conservation
* Non-global logarithms
* Colour

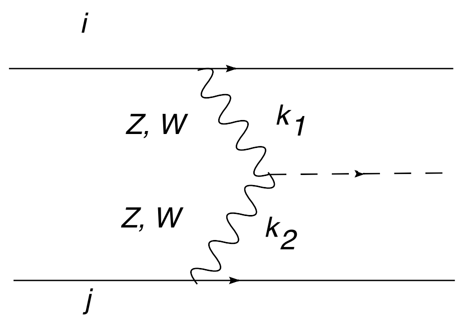
<u>Very significant contribution from Coulomb gluons heralds the breakdown of the angular ordered parton shower approach and failure of the standard parton showers (Pythia, Herwig, Sherpa)</u>

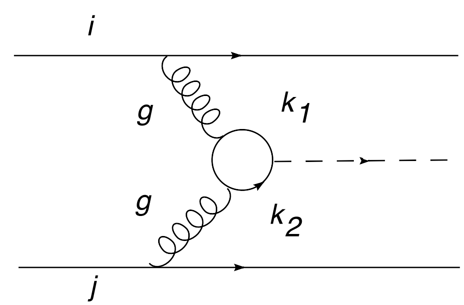


G. Aad et al. [ATLAS Collaboration], [arXiv:1107.1641 [hep-ex]].
Plots from Delgado, JF, Marzani & Seymour – JHEP 2011.

Theory needs improving…..

Jet vetoing as a tool: Probing Higgs couplings





- To reduce backgrounds and to focus on the VBF channel, experimenters will make a veto on additional radiation between the tag jets, i.e. no additional jets with

$$k_T \geq Q_0$$

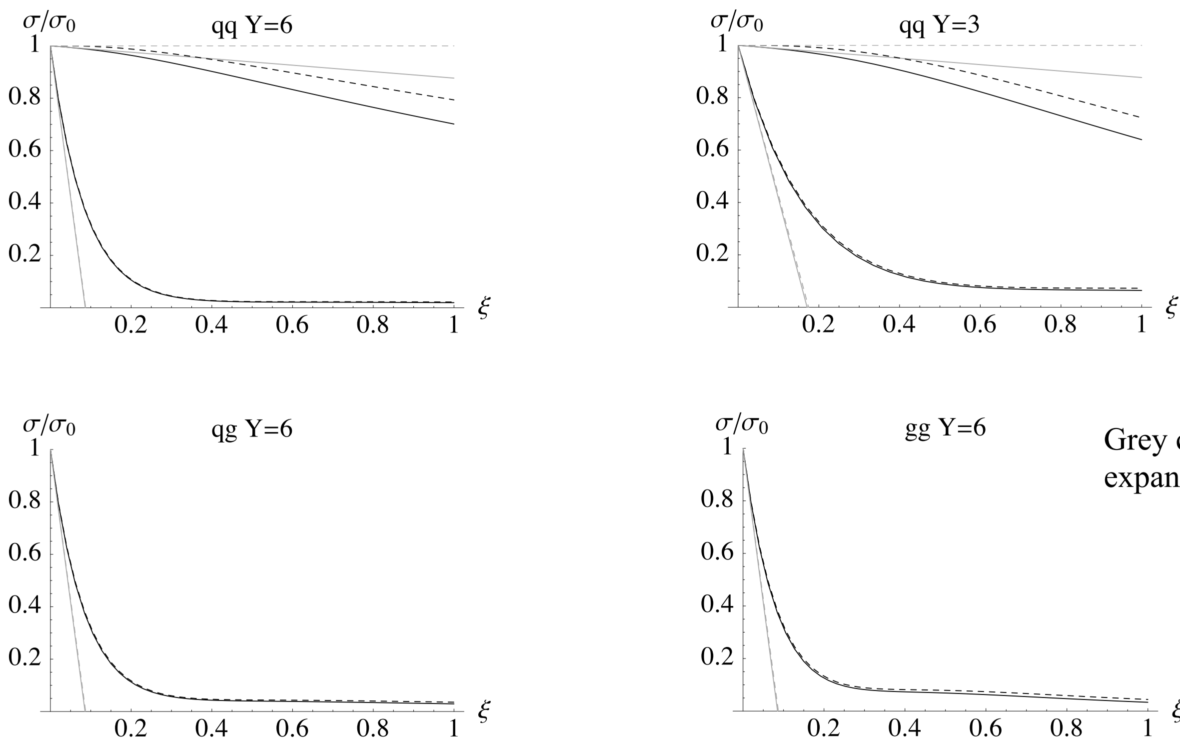
- Soft gluon effects will induce logarithms:

$$\alpha_s^n \ln^n (Q/Q_0)$$

$Q =$ transverse momentum of tag jets

Resummation proceeds almost exactly as for “gaps between jets”

$$\mathbf{M} = \exp\left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{dk_T}{k_T} \, \mathbf{\Gamma} \right) \mathbf{M}_0 \qquad \rho(Y, \Delta y) \to \frac{1}{2}(\rho(Y, 2|y_3|) + \rho(Y, 2|y_4|))$$



Grey curves = lowest order expansion of black curves.

Only the colour of the exchange matters.

$$\xi = \int_{Q_0}^{Q} \frac{dq}{q} \alpha_s(q) \approx 0.2$$ for 100 GeV jets and a 20 GeV veto, i.e. resummation is important at LHC

JF & Malin Sjödahl (2007)

- Fixed-order calculations may/will not account adequately for the effect of a veto.
  Especially for gluon fusion.

- We know much of this physics is not included in the parton shower Monte Carlos.
  Subleading N_c and Coulomb gluon contributions and other colour mixing effects.

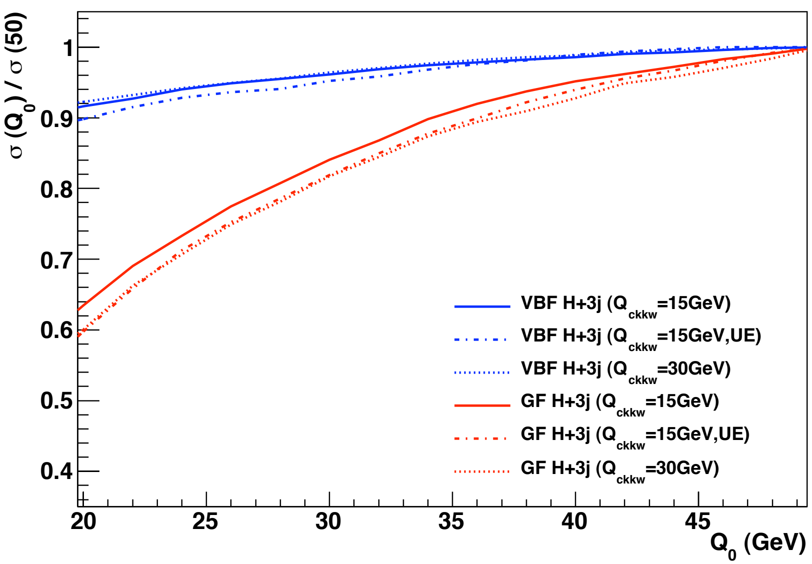
Extracting the Higgs' couplings

Focus on Higgs decay to tau pairs

$$\sigma(Q_0) = \Lambda_g \sigma_g^{SM}(Q_0) + \Lambda_V \sigma_V^{SM}(Q_0)$$

$$\Lambda_g \sigma_g^{SM} \propto \frac{\Gamma_{gg}\Gamma_{\tau\tau}}{\Gamma_T}$$

$$\Lambda_V \sigma_V^{SM} \propto \frac{\Gamma_{VV}\Gamma_{\tau\tau}}{\Gamma_T}$$



Curves obtained using Sherpa with CKKW matching

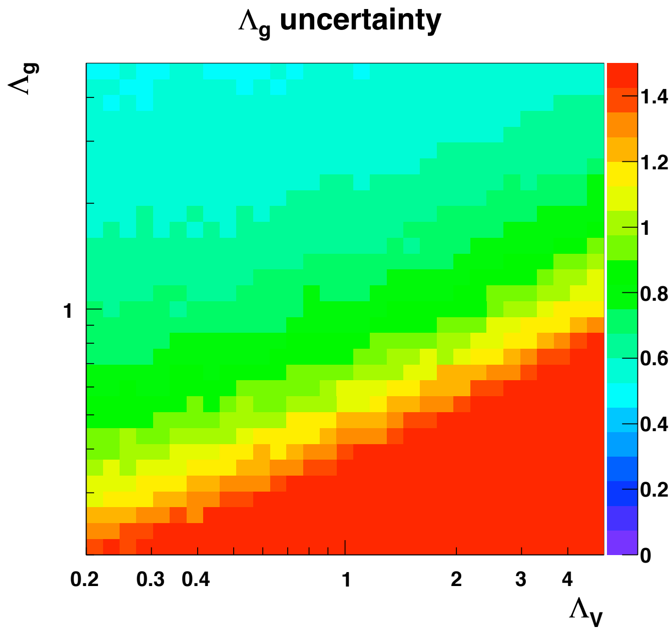
How uncertain are the theoretical predictions?

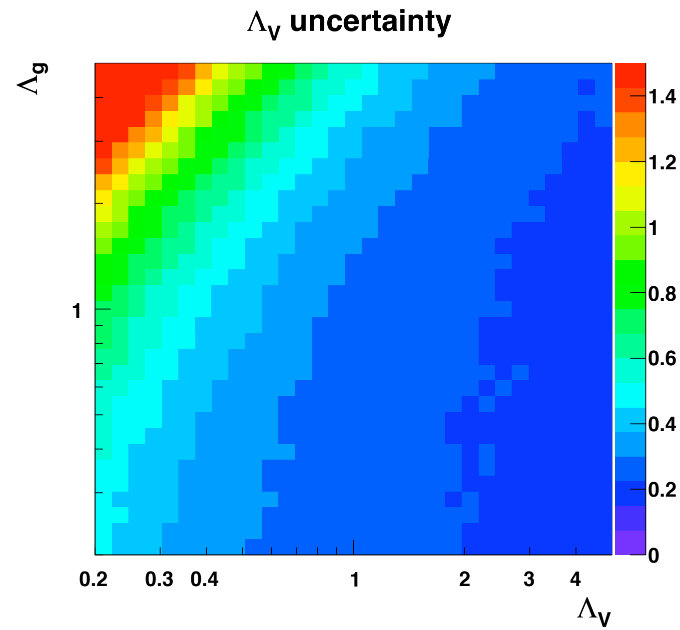
$$\sigma(Q_0) = \sigma_{jj}(1 - P_{veto}(Q_0))$$

VBF
σ_{jj} known to $\pm 4\%$
(Full NLO: Ciccolini, Denner, Dittmaier. Partial NNLO: Bolzoni, Maltoni, Moch, Zaro)
P_{veto} known to $\pm 1\%$
(Estimated NLO $Hjjj$: Figy, Hankele, Zeppenfeld)

GF
σ_{jj} known to $\pm 20\%$
(Full NLO: Campbell, Ellis, Zanderighi)
P_{veto} known to $\pm 20\%$
(May be larger at present, e.g. see Andersen et al, Les Houches 2010)

B. Cox, JF & A. Pilkington Physics Letters B (2010)





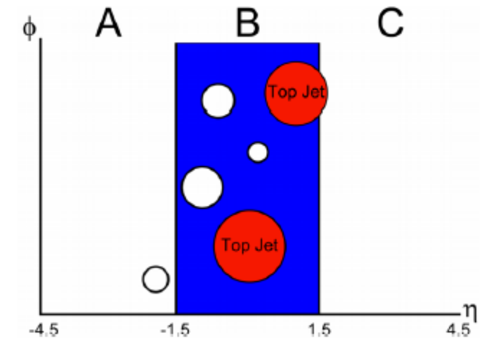
60/fb

14 TeV

Error	SM ($\Lambda_{g,V} = 1$) $\sigma_{\Lambda_g}/\Lambda_g$	SM ($\Lambda_{g,V} = 1$) $\sigma_{\Lambda_V}/\Lambda_V$	BSM ($\Lambda_g = 4$, $\Lambda_V = 1/4$) $\sigma_{\Lambda_g}/\Lambda_g$	BSM ($\Lambda_g = 4$, $\Lambda_V = 1/4$) $\sigma_{\Lambda_V}/\Lambda_V$
Stat. only	0.51 [0.23]	0.16 [0.07]	0.19 [0.08]	0.72 [0.33]
Backgd.	0.56 [0.25]	0.18 [0.08]	0.20 [0.09]	0.79 [0.35]
VBF	0.52 [0.25]	0.17 [0.08]	0.19 [0.08]	0.75 [0.33]
GF	0.65 [0.45]	0.19 [0.11]	0.43 [0.40]	1.56 [1.40]
Expt.	0.62 [0.39]	0.26 [0.21]	0.35 [0.31]	0.89 [0.52]
All	0.77 [0.57]	0.28 [0.23]	0.53 [0.50]	1.66 [1.49]

Key challenge is to understand the theory of jet vetoing better

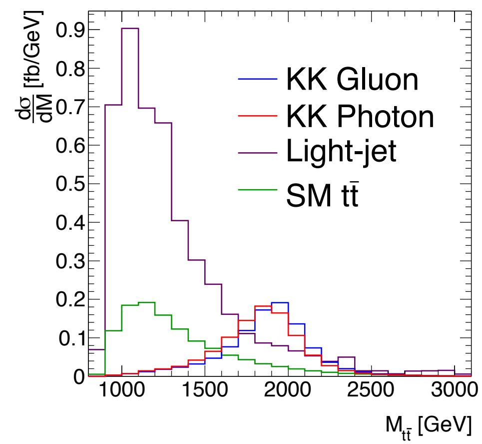
Jet vetoing as a tool: TeV-scale resonances



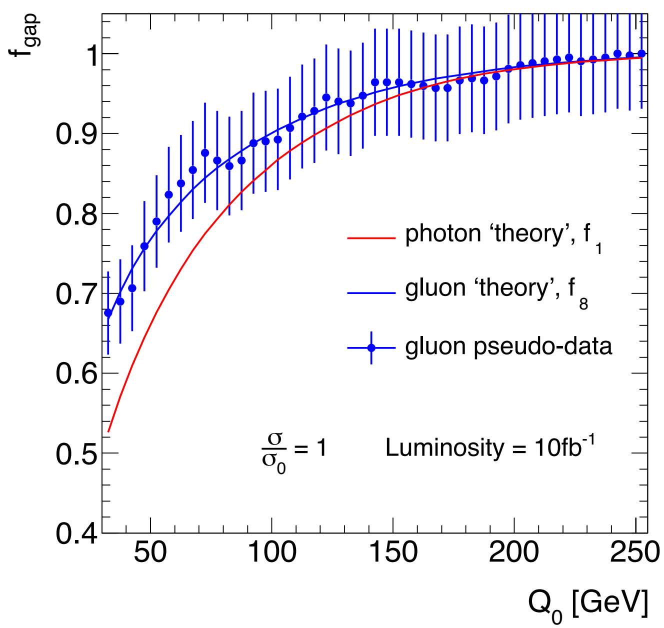
Consider 2 TeV colour-singlet and colour-octet resonances decaying to highly-boosted top-antitop pairs.

Typical of KK states in RS models.

Added as a new process in Pythia 8.



$$f_{gap}(Q_0) = a_1\, f_1(Q_0) + a_8\, f_8(Q_0)$$

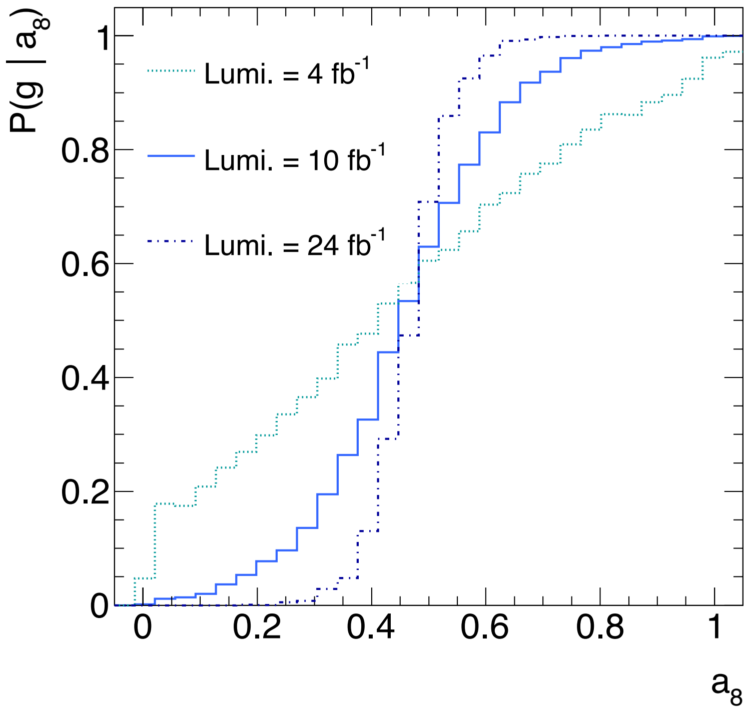


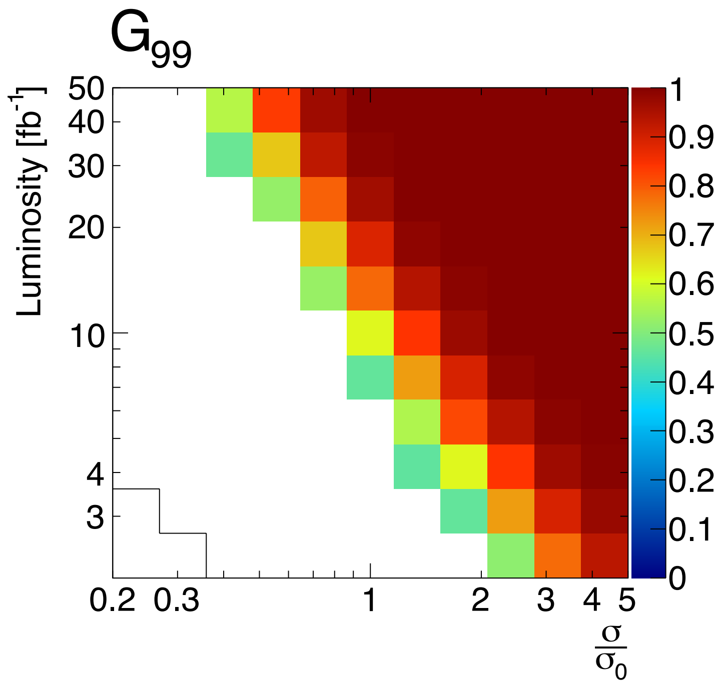
- Theory uncertainty on signal used in fit => major challenge
- Background can be inferred from data.
  Note: can interfere with signal in a manner dependent upon the new physics model. Only a "problem" if it is Q0 dependent.
- Experimental systematics: Jet energy resolution but will be small compared to statistical uncertainty (as in ATLAS dijet measurement).
- Only analyse when signal is > 5 sigma significant

- Canonical RS scenario: All couplings vector-like and production cross-section 1 pb before any cuts.
  Coupling to light quarks and top only (no coupling to gluons).
  Mass = 2 TeV and width = 400 GeV.
- Background from QCD dijets and top-pairs.
- Use Johns Hopkins top tagger algorithm (Kaplan et al)
- Veto on jets using anti-kT algorithm with R=0.6

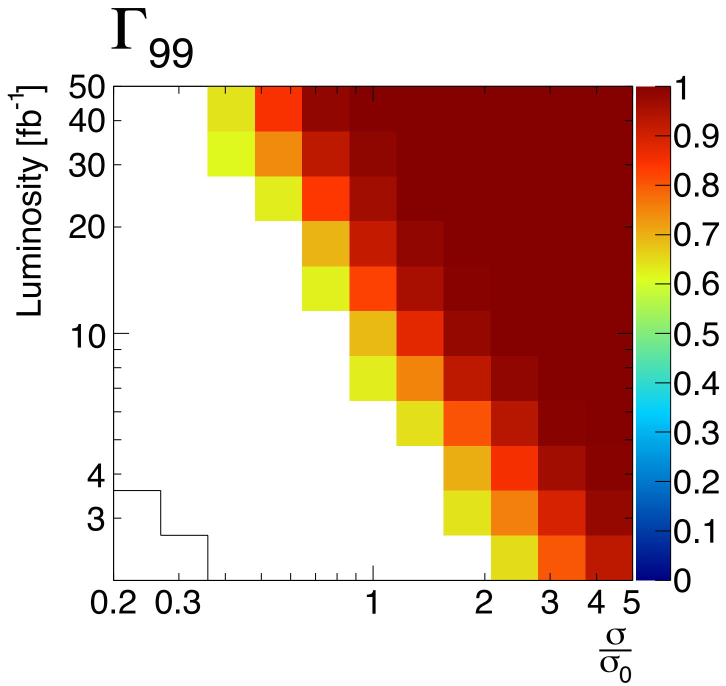
S. Ask, J.H. Collins, JF, A. Pilkington, arXiv:1108.2396 [hep-ph]. See also Ilmo Sung (Phys.Rev. D80 (2009) 094020).

$$P(\mathrm{g}|a_8) = \frac{P(a_8|\mathrm{g})}{P(a_8|\mathrm{g}) + P(a_8|\gamma)}$$



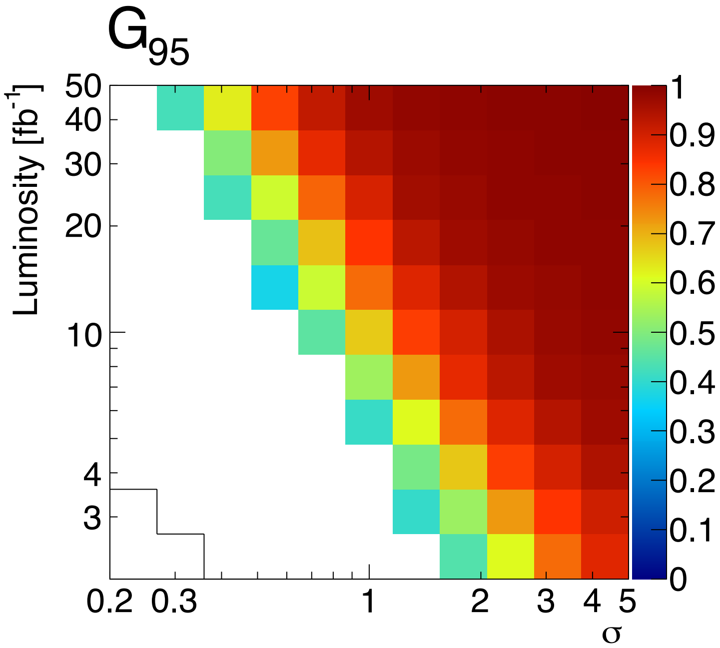


The fraction of LHC experiments that would Measure P(gluon|a8) > 99% assuming a HEAVY GLUON

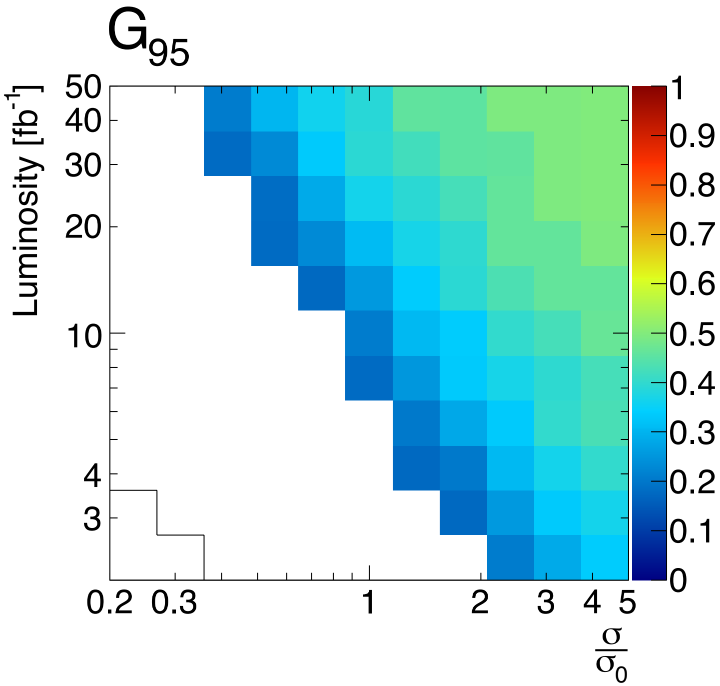


The fraction of LHC experiments that would Measure P(photon|a1) > 99% assuming a HEAVY PHOTON

Ignoring theory uncertainty



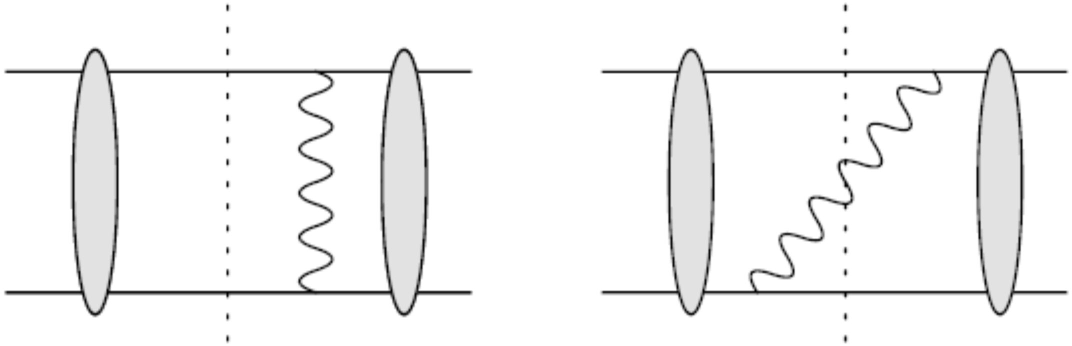
The fraction of LHC experiments that would Measure P(gluon|a8) > 95% assuming a HEAVY GLUON and 10% theory error.



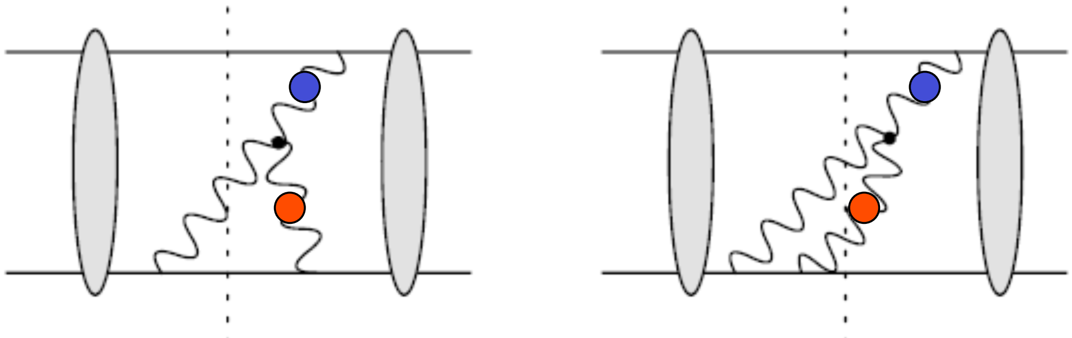
The fraction of LHC experiments that would Measure P(gluon|a8) > 95% assuming a HEAVY GLUON and a 25% theory error.

Including theory uncertainty

But the theory is not just "Sudakov": these observables are *non-global*



Such real & virtual corrections cancel.



But these do not if the gluon marked with a red blob is in the forbidden region: the 2nd cut is not allowed.

So the cancellation does not hold…..

real and virtual

It fails only once we start to evolve emissions (such as those denoted by the blue blob in the above) which lie *outside of the gap* region and which have $k_T > Q_0$

$|y| > Y/2$

If $k_T < Q_0$ then subsequent evolution also has $k_T < Q_0$ and cancellation works.

- The miscancellation is telling us that this observable is sensitive to soft gluon emissions outside of the gap, even though the observable sums inclusively over that region.

- Not a surprise once we realise that emissions outside of the gap can subsequently radiate back into the gap.

- We must therefore include any number of emissions outside of the gap and their subsequent evolution.

- Colour structure makes this impossible using current technology.

- We could aim to compute the all-orders non-global corrections in the leading N_c approximation. Dasgupta, Salam, Appleby, Seymour, Delenda, Banfi

- Instead we shall compute the "one hard emission out of the gap" contribution without any approximation on the colour.

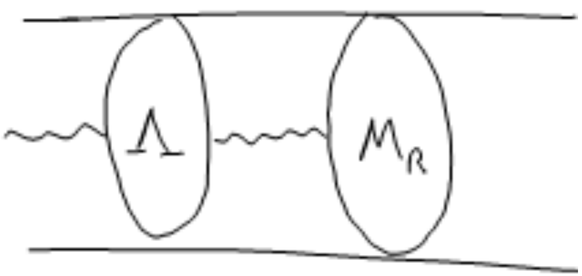
Two new ingredients still sticking to quark-quark scattering

1) How to add a real gluon to the four-parton amplitude

$$\mathbf{M}_R = \mathbf{D} \cdot \mathbf{M}$$

2) How to evolve the resulting five-parton amplitude

$$\mathbf{M}_R(Q_0) = \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\,\mathbf{\Lambda}\right)\mathbf{M}_R(k_T)$$



Kyrieleis & Seymour

$$\mathbf{D}^{\mu} = \begin{pmatrix} \frac{1}{2}(-h_1^{\mu} - h_2^{\mu} + h_3^{\mu} + h_4^{\mu}) & \frac{1}{4N}(-h_1^{\mu} - h_2^{\mu} + h_3^{\mu} + h_4^{\mu}) \\ 0 & \frac{1}{2}(-h_1^{\mu} - h_2^{\mu} + h_3^{\mu} + h_4^{\mu}) \\ \frac{1}{2}(-h_1^{\mu} + h_2^{\mu} + h_3^{\mu} - h_4^{\mu}) & \frac{1}{4N}(h_1^{\mu} - h_2^{\mu} - h_3^{\mu} + h_4^{\mu}) \\ 0 & \frac{1}{2}(-h_1^{\mu} + h_2^{\mu} - h_3^{\mu} + h_4^{\mu}) \end{pmatrix} \qquad h_i^{\mu} = \frac{1}{2} k_T \frac{p_i^{\mu}}{p_i \cdot k}$$

$$\mathbf{\Lambda} = \begin{pmatrix} \frac{N}{4}(Y - i\pi) + \frac{1}{2N} i\pi & \left(\frac{1}{4} - \frac{1}{N^2}\right) i\pi & -\frac{N}{4} s_y Y & 0 \\ i\pi & \frac{N}{4}\left(2Y - i\pi\right) - \frac{3}{2N} i\pi & 0 & 0 \\ -\frac{N}{4} s_y Y & 0 & \frac{N}{4}(Y - i\pi) - \frac{1}{2N} i\pi & -\frac{1}{4} i\pi \\ 0 & 0 & -i\pi & \frac{N}{4}\left(2Y - i\pi\right) - \frac{1}{2N} i\pi \end{pmatrix}$$

$$+ \begin{pmatrix} N & 0 & 0 & 0 \\ 0 & N & 0 & 0 \\ 0 & 0 & N & 0 \\ 0 & 0 & 0 & N \end{pmatrix} \frac{1}{4} \rho(Y, 2\,|y|)$$

$$+ \begin{pmatrix} C_F & 0 & 0 & 0 \\ 0 & C_F & 0 & 0 \\ 0 & 0 & C_F & 0 \\ 0 & 0 & 0 & C_F \end{pmatrix} \frac{1}{2} \rho(Y, \Delta y)$$

$$+ \begin{pmatrix} \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & 0 & \frac{N}{4}\left(-\frac{1}{2} s_y \lambda\right) & \frac{1}{4}\left(\frac{1}{2} s_y \lambda\right) \\ 0 & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & 0 & \frac{N}{4}\left(\frac{1}{2} s_y \lambda\right) \\ \frac{N}{4}\left(-\frac{1}{2} s_y \lambda\right) & 0 & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & \frac{1}{4}\left(-\frac{1}{2}\lambda\right) \\ \frac{1}{2} s_y \lambda & \left(\frac{N}{4} - \frac{1}{N}\right)\left(\frac{1}{2} s_y \lambda\right) & -\frac{1}{2}\lambda & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) \end{pmatrix}$$

Has been extended to all five parton amplitudes:

e.g. gg → ggg

$$\left(\begin{array}{cccccccccccccccc}
-\frac{1}{2} N (k_{S}+k_{S5}) & \frac{N k_{dds}}{-1+N^2} & \frac{N^2(-2k_{125}+k_{ssd})}{2(-1+N^2)} & \frac{(-4+N^2)k_{dds}}{2(-1+N^2)} & 0 & 0 & 0 & 0 & -\frac{N^3 k_{dds}}{(-1+N^2)^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{N k_{dds}}{-1+N^2} & -\frac{1}{2} N (k_{O5}+k_{OS}) & \frac{N^2(2k_{345}-k_{dds})}{2(-1+N^2)} & 0 & \frac{(-4+N^2)k_{dds}}{2(-1+N^2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{N^3 k_{dds}}{(-1+N^2)^2} & 0 & 0 \\
\frac{1}{2}(-2k_{125}+k_{ssd}) & k_{345}-\frac{k_{dds}}{2} & -\frac{1}{8}N(2k_{O5}+k_{OO}+2k_{S5}) & -\frac{(-4+N^2)(-2k_{345}+k_{dds})}{8N} & \frac{(-4+N^2)(-2k_{125}+k_{ssd})}{8N} & \frac{(-4+N^2)k_{dds}}{8N} & 0 & 0 & \frac{(-3+N^2)(2k_{345}-k_{dds})}{4(-1+N^2)} & 0 & 0 & 0 & 0 & -\frac{(-3+N^2)(2k_{125}-k_{ssd})}{4(-1+N^2)} & 0 & -\frac{k_{dds}}{4} \\
\frac{N^2 k_{dds}}{2(-4+N^2)} & 0 & \frac{N^3(2k_{345}-k_{dds})}{8(-4+N^2)} & -\frac{1}{8}N(2k_{O5}+k_{OO}+2k_{S5}) & \frac{N(-12+N^2)k_{dds}}{8(-4+N^2)} & \frac{1}{8}N(-2k_{125}+k_{ssd}) & \frac{N^2(-2k_{345}+k_{dds})}{4(-4+N^2)} & \frac{N k_{dds}}{8-2N^2} & 0 & 0 & 0 & 0 & 0 & -\frac{N^4(-7+N^2)k_{dds}}{4(-4+N^2)^2(-1+N^2)} & \frac{N^2(-6+N^2)k_{dds}}{4(-4+N^2)^2} & 0 \\
0 & \frac{N^2 k_{dds}}{2(-4+N^2)} & \frac{N^3(-2k_{125}+k_{ssd})}{8(-4+N^2)} & \frac{N(-12+N^2)k_{dds}}{8(-4+N^2)} & -\frac{1}{8}N(2k_{O5}+k_{OO}+2k_{S5}) & \frac{1}{8}N(2k_{345}-k_{dds}) & 0 & 0 & -\frac{N^4(-7+N^2)k_{dds}}{4(-4+N^2)^2(-1+N^2)} & \frac{N^2(2k_{125}+k_{ssd})}{4(-4+N^2)} & \frac{N k_{dds}}{8-2N^2} & 0 & \frac{N^2(-6+N^2)k_{dds}}{4(-4+N^2)^2} & 0 & 0 & 0 \\
0 & 0 & \frac{N^3 k_{dds}}{8(-4+N^2)} & \frac{1}{8}N(-2k_{125}+k_{ssd}) & \frac{1}{8}N(2k_{345}-k_{dds}) & -\frac{1}{8}N(2k_{O5}+k_{OO}+2k_{S5}) & \frac{N^2 k_{dds}}{4(-4+N^2)} & 0 & 0 & \frac{N^2 k_{dds}}{4(-4+N^2)} & 0 & \frac{N^2 k_{dds}}{16-4N^2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}(-2k_{345}+k_{dds}) & 0 & \frac{k_{dds}}{2} & -\frac{1}{4}N(k_{OD}+2k_{S5}) & \frac{1}{4}(-2k_{125}+k_{ssd}) & -\frac{N(-6+N^2)(2k_{345}-k_{dds})}{4(-4+N^2)} & 0 & 0 & 0 & 0 & 0 & \frac{N(-2k_{125}+k_{ssd})}{4(-4+N^2)} & \frac{N k_{dds}}{4(-4+N^2)} \\
0 & 0 & 0 & -\frac{N k_{dds}}{-4+N^2} & 0 & 0 & \frac{N^2(-2k_{125}+k_{ssd})}{4(-4+N^2)} & -\frac{1}{4}N(k_{OD}+2k_{S5}) & \frac{N^4 k_{dds}}{4(-4+N^2)^2} & 0 & \frac{N k_{dds}}{2(-4+N^2)} & \frac{N^2(-2k_{125}+k_{ssd})}{4(-4+N^2)} & -\frac{N^2(-6+N^2)k_{dds}}{4(-4+N^2)^2} & 0 & 0 & 0 \\
-\frac{2N k_{dds}}{-3+N^2} & 0 & k_{345}-\frac{k_{dds}}{2} & 0 & -\frac{(-7+N^2)k_{dds}}{2(-3+N^2)} & 0 & \frac{(6-7N^2+N^4)(-2k_{345}+k_{dds})}{4N(-3+N^2)} & \frac{(-1+N^2)k_{dds}}{4(-3+N^2)} & \frac{N(k_{O27}+4k_{O5}-2(-1+N^2)k_{S5})}{4(-3+N^2)} & 0 & 0 & 0 & 0 & \frac{2(3-7N^2+N^4)k_{dds}}{N(-12+19N^2-8N^4+N^6)} & -\frac{(8-9N^2+N^4)k_{dds}}{4N(12-7N^2+N^4)} & \frac{3(-1+N^2)(-2k_{125}+k_{ssd})}{4N(-3+N^2)} \\
0 & 0 & 0 & 0 & k_{125}+\frac{k_{ssd}}{2} & \frac{k_{dds}}{2} & 0 & 0 & 0 & -\frac{1}{4}N(k_{DO}+2k_{O5}) & \frac{1}{4}(2k_{345}-k_{dds}) & 0 & \frac{N(2k_{345}-k_{dds})}{4(-4+N^2)} & \frac{N(-6+N^2)(2k_{125}-k_{ssd})}{4(-4+N^2)} & 0 & \frac{N k_{dds}}{4(-4+N^2)} \\
0 & 0 & 0 & 0 & -\frac{N k_{dds}}{-4+N^2} & 0 & 0 & \frac{N k_{dds}}{2(-4+N^2)} & 0 & \frac{N^2(2k_{345}-k_{dds})}{4(-4+N^2)} & -\frac{1}{4}N(k_{DO}+2k_{O5}) & \frac{N^2(2k_{345}-k_{dds})}{4(-4+N^2)} & 0 & \frac{N^4 k_{dds}}{4(-4+N^2)^2} & -\frac{N^2(-6+N^2)k_{dds}}{4(-4+N^2)^2} & 0 \\
0 & 0 & 0 & 0 & 0 & -k_{dds} & 0 & \frac{1}{2}(-2k_{125}+k_{ssd}) & 0 & 0 & k_{345}-\frac{k_{dds}}{2} & -\frac{1}{8}N(2(k_{O5}+k_{S5})+k_{D,D}) & -\frac{N(-8+N^2)(2k_{345}-k_{dds})}{8(-4+N^2)} & 0 & \frac{N(-8+N^2)(2k_{125}-k_{ssd})}{8(-4+N^2)} & \frac{N(-16+3N^2)k_{dds}}{8(-4+N^2)} \\
0 & 0 & 0 & 0 & k_{dds} & 0 & 0 & -\frac{k_{dds}}{2} & 0 & -\frac{N(-2k_{345}+k_{dds})}{2(-6+N^2)} & 0 & \frac{N(-8+N^2)(-2k_{345}+k_{dds})}{8(-6+N^2)} & \frac{N(-2(-8+N^2)k_{O5}-2(-4+N^2)k_{S5}+k_{D,27})}{8(-6+N^2)} & -\frac{N(-8+N^2)k_{dds}}{2(24-10N^2+N^4)} & \frac{(-8+N^2)(24-20N^2+3N^4)k_{dds}}{8N(24-10N^2+N^4)} & -\frac{N(-4+N^2)(-2k_{125}+k_{ssd})}{8(-6+N^2)} \\
0 & -\frac{2N k_{dds}}{-3+N^2} & \frac{1}{2}(-2k_{125}+k_{ssd}) & -\frac{(-7+N^2)k_{dds}}{2(-3+N^2)} & 0 & 0 & 0 & 0 & \frac{2(3-7N^2+N^4)k_{dds}}{N(-12+19N^2-8N^4+N^6)} & -\frac{(6-7N^2+N^4)(-2k_{125}+k_{ssd})}{4N(-3+N^2)} & \frac{(-1+N^2)k_{dds}}{4(-3+N^2)} & 0 & -\frac{(8-9N^2+N^4)k_{dds}}{4N(12-7N^2+N^4)} & \frac{N(4k_{S5}-2(-1+N^2)(\Omega_{15}+\Omega_{25})+k_{27,O})}{4(-3+N^2)} & 0 & -\frac{3(-1+N^2)(-2k_{345}+k_{dds})}{4N(-3+N^2)} \\
0 & 0 & 0 & k_{dds} & 0 & 0 & \frac{N(-2k_{125}+k_{ssd})}{2(-6+N^2)} & 0 & -\frac{N(-8+N^2)k_{dds}}{2(24-10N^2+N^4)} & 0 & -\frac{k_{dds}}{2} & -\frac{N(-8+N^2)(-2k_{125}+k_{ssd})}{8(-6+N^2)} & \frac{(-8+N^2)(24-20N^2+3N^4)k_{dds}}{8N(24-10N^2+N^4)} & 0 & \frac{N(k_{27D}-2((-4+N^2)k_{O5}+(-8+N^2)k_{S5}))}{8(-6+N^2)} & \frac{N(-4+N^2)(-2k_{345}+k_{dds})}{8(-6+N^2)} \\
0 & 0 & -k_{dds} & 0 & 0 & 0 & \frac{k_{dds}}{2N} & 0 & \frac{3(-2k_{125}+k_{ssd})}{2N} & \frac{k_{dds}}{2N} & 0 & \left(-\frac{2}{N}+\frac{3N}{8}\right)k_{dds} & -\frac{(-4+N^2)(-2k_{125}+k_{ssd})}{8N} & -\frac{3(-2k_{345}+k_{dds})}{2N} & \frac{(-4+N^2)(-2k_{345}+k_{dds})}{8N} & -\frac{k_{2727a}+2N^2(k_{O5}+k_{S5})}{8N}
\end{array}\right)$$

Sjödahl (2008)

….for an arbitrary n-parton amplitude:

$$\mathbf{\Gamma} = -\sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \; \Omega_{ij}$$

$$\Omega_{ij} = \frac{1}{2}\left\{ \int_{\rm veto} \frac{dy\, d\phi}{2\pi} \frac{1}{2} k_T^2 \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)} - i\pi\, \Theta(ij = II \text{ or } FF) \right\}$$

$$\begin{aligned} \mathbf{\Gamma} \;&=\; \frac{1}{2} Y \mathbf{T}_t^2 + i\pi \mathbf{T}_1 \cdot \mathbf{T}_2 + \frac{1}{4} \sum_{i \in F} \rho(Y; 2|y_i|) \mathbf{T}_i^2 \\ &+ \frac{1}{2} \sum_{(i<j) \in L} \lambda(Y; |y_i| + |y_j|, |\phi_i - \phi_j|) \mathbf{T}_i \cdot \mathbf{T}_j + \frac{1}{2} \sum_{(i<j) \in R} \lambda(Y; |y_i| + |y_j|, |\phi_i - \phi_j|) \mathbf{T}_i \cdot \mathbf{T}_j \;. \end{aligned}$$

Easy to see it is final state collinear safe but not initial state collinear safe:

i.e. $\mathbf{\Gamma} \sim \mathbf{T}_i + \mathbf{T}_j$ only for *i* and *j* collinear *and* in final state

A surprise: True beyond one-loop (massless case):
2-loop: Mert Aybat, Dixon, Sterman (2006)
3-loop: Dixon (2009)
n-loop? Bern et al (2009), Gardi & Magnea (2009), Becher & Neubert (2009)

Failure at 2-loop in massive case…
Mitov, Sterman & Sung (2009)

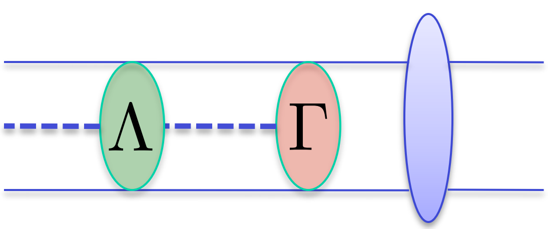
JF, Kyrieleis & Seymour (2008)

The complete cross-section for one real emission outside of the gap is thus

$$\sigma_R = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_{\text{out}}\frac{dy\,d\phi}{2\pi}$$

$$\mathbf{M}_0^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\boldsymbol{\Gamma}^\dagger\right)\mathbf{D}_\mu^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\boldsymbol{\Lambda}^\dagger\right)\mathbf{S}_R$$

$$\exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\boldsymbol{\Lambda}\right)\mathbf{D}^\mu \exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\boldsymbol{\Gamma}\right)\mathbf{M}_0$$



And the corresponding contribution when the out-of-gap gluon is virtual is

$$\sigma_V = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_{\text{out}}\frac{dy\ d\phi}{2\pi}$$

$$\left[\mathbf{M}_0^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}^\dagger\right)\mathbf{S}_V\right.$$

$$\left.\exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\mathbf{\Gamma}\right)\gamma\exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}\right)\mathbf{M}_0 + \text{c.c.}\right]$$

Adds one "out of the gap" virtual gluon

Conventional wisdom: when the out of gap gluon becomes collinear with either incoming quark or either outgoing quark the real and virtual contributions should cancel.

This cancellation operates for **final state collinear emission**:

$$\mathbf{D}^{\mu\dagger}(\mathbf{\Lambda}^{\dagger})^{n-m}\mathbf{S}_R\mathbf{\Lambda}^m\mathbf{D}_\mu + (\mathbf{\Gamma}^{\dagger})^{n-m}\mathbf{S}_V\mathbf{\Gamma}^m\gamma + \gamma^{\dagger}(\mathbf{\Gamma}^{\dagger})^{n-m}\mathbf{S}_V\mathbf{\Gamma}^m = \mathbf{0}$$

But it fails for **initial state collinear emission**:

The problem is entirely due to the emission of Coulomb gluons.

Cancellation *does* occur for $n = 1$, 2 and 3 gluons relative to lowest order but not for larger n. This is the lowest order where the Coulomb gluons do not trivially cancel.

What are we to make of a non-cancelling collinear divergence?

$$\sigma \sim \sigma_0 \; \alpha^4 L^4 \pi^2 Y \int_{\text{out}} dy$$

Cannot actually have infinite rapidity with $k_T > Q_0$

Need to go beyond soft gluon approximation in collinear (large rapidity) limit:

$$\int d^2k_T \int_{\text{out}} dy \;\; \frac{d\sigma}{dy d^2k_T}\bigg|_{\text{soft}} \rightarrow \int d^2k_T \left[\int^{y_{\max}} dy \;\; \frac{d\sigma}{dy d^2k_T}\bigg|_{\text{soft}} + \int_{y_{\max}}^{\infty} dy \;\; \frac{d\sigma}{dy d^2k_T}\bigg|_{\text{collinear}} \right]$$

$$\int_{y_{\max}}^{\infty} dy \;\; \frac{d\sigma}{dy d^2k_T}\bigg|_{\text{collinear}} = \int_{y_{\max}}^{\infty} dy \; \left(\frac{d\sigma_{\text{R}}}{dy d^2k_T}\bigg|_{\text{collinear}} + \frac{d\sigma_{\text{V}}}{dy d^2k_T}\bigg|_{\text{collinear}} \right)$$

Soft approximation:

$$\int dz \, \frac{1}{2}\left(\frac{1+z^2}{1-z}\right) \to \int dy$$

Real collinear emission:

$$\int_{y_{\max}}^{\infty} dy \; \frac{d\sigma_{\rm R}}{dy d^2 k_T}\bigg|_{\rm collinear} = \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right)\frac{q(x/z,\mu^2)}{q(x,\mu^2)} A_{\rm R}$$

$$= \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right)\left(\frac{q(x/z,\mu^2)}{q(x,\mu^2)} - 1\right) A_{\rm R} + \int_0^{1-\delta} dz \frac{1}{2}\frac{1+z^2}{1-z} A_{\rm R}$$

Virtual collinear emission:

$$\int_{y_{\max}}^{\infty} dy \; \frac{d\sigma_{\rm V}}{dy d^2 k_T}\bigg|_{\rm collinear} = \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right) A_{\rm V}$$

$$y > y_{\max}$$

implies

$$\delta \approx \frac{k_T}{Q} \exp\left(y_{\max} - \frac{\Delta y}{2}\right)$$

If $A_{\rm R} + A_{\rm V} = 0$ then the divergence would cancel leaving behind a regularized splitting, which would correspond to the DGLAP evolution of the incoming quark pdf.

But as we have seen, the Coulomb gluons spoil this cancellation. Instead we have

$$\int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right)(A_{\rm R}+A_{\rm V}) = \ln\left(\frac{1}{\delta}\right)(A_{\rm R}+A_{\rm V}) + \text{subleading}$$

$$\approx \left(-y_{\max} + \frac{\Delta y}{2} + \ln\left(\frac{Q}{k_T}\right)\right)(A_{\rm R}+A_{\rm V})$$

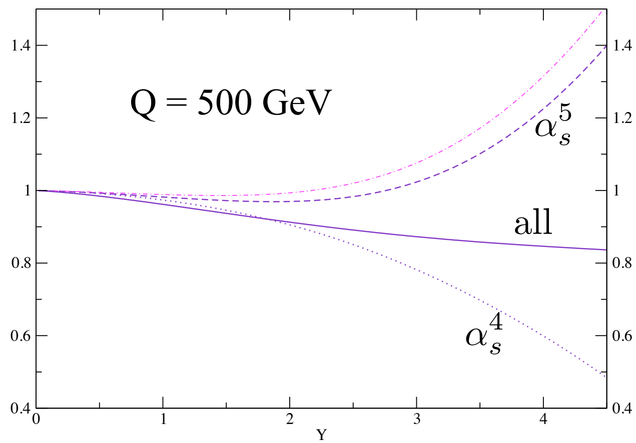
Hence

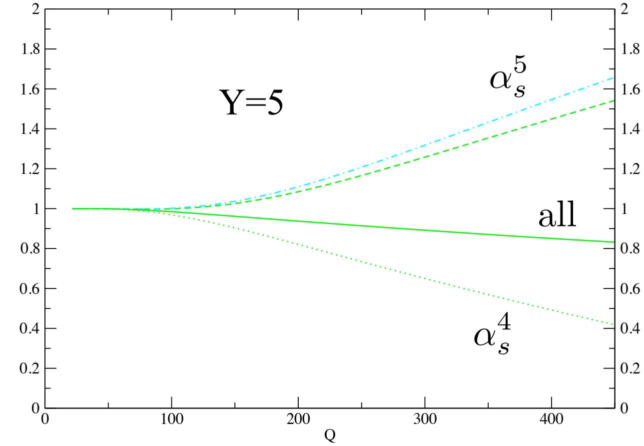
$$\int_{Q_0}^{Q} \frac{dk_T}{k_T} \int_{\text{out}} dy \to \int_{Q_0}^{Q} \frac{dk_T}{k_T}\left(\int^{y_{\max}} dy + \left(-y_{\max} + \ln\frac{Q}{k_T}\right)\right) = \frac{1}{2}\ln^2\frac{Q}{Q_0}$$

The final result for the "one emission out-of-gap" cross-section is

$$\sigma_{1,\rm SLL} = -\sigma_0 \left(\frac{2\alpha_s}{\pi}\right)^4 \ln^5\left(\frac{Q}{Q_0}\right)\pi^2 Y\, \frac{(3N^2-4)}{480}$$

Now an estimate of the impact due to super-leading logarithms…
K-factors relative to “primary emissions only” prediction.





Upper curve at $O(\alpha_s^5)$ includes the contribution from two “out-of-gap” gluons

Less than 20% effect for Q < 500 GeV and Y < 5

Conclusions

- Wide-angle soft gluons are not very well understood/simulated.
- Understanding them can be exploited to learn about new physics.
- Existence of super-leading logarithms = Breakdown of QCD coherence = Failure of collinear factorization = intriguing.
- Measurements at LHC on jet vetoing in (i) dijet events; (ii) W/Z + n jets (n = 2 especially interesting); (iii) Higgs + dijets; (iv) Other new physics events, e.g. top pairs.