Studies of *b* quark decays using experiment plus lattice QCD

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Outline

- Quark flavour & Lattice QCD
- DiRAC facility
- Example: $|V_{cb}|$ from $B \to D^* l \nu$

Quark Flavour
&
Lattice QCD

Motivation

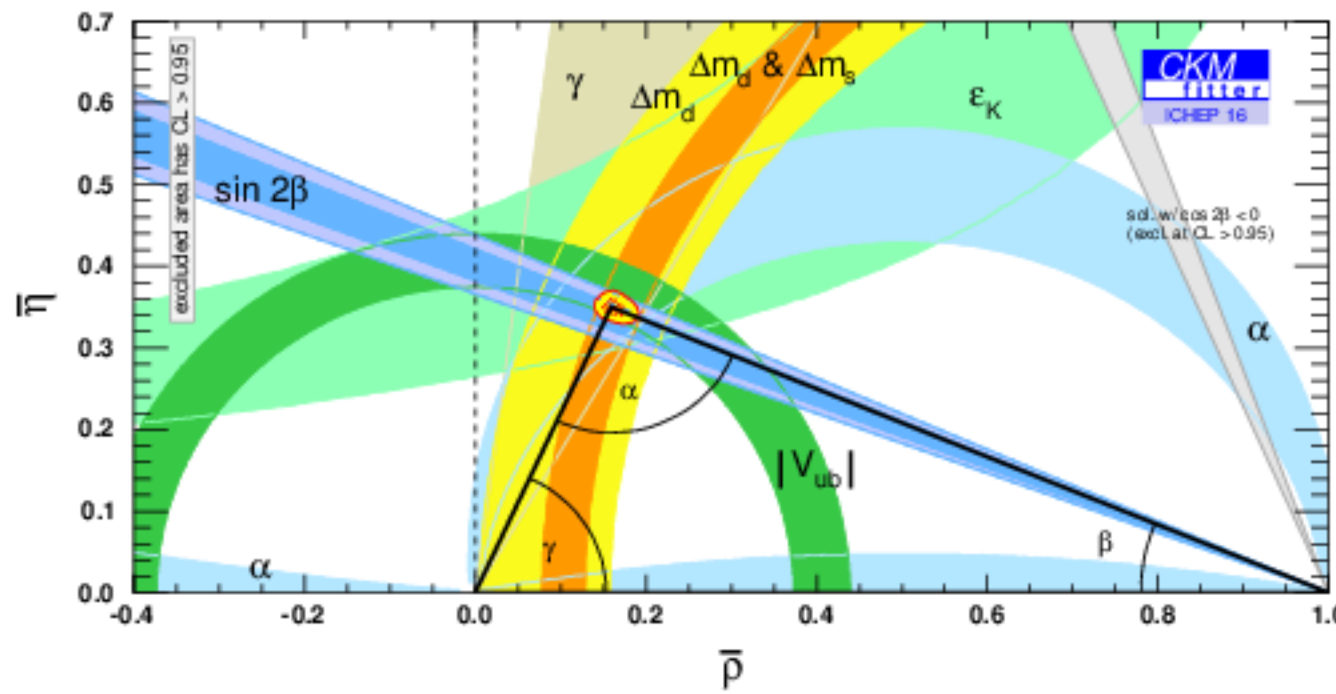
- Precision predictions & measurements of quark flavour interactions
- Is the Standard Model description of EWSB complete?
- If not, quark flavour measurements constrain models of new physics
- Experimental measurements of hadron decays: increasing precision, new modes
- Precision QCD calculations required in order to make inferences about quark interactions

Quark flavour physics

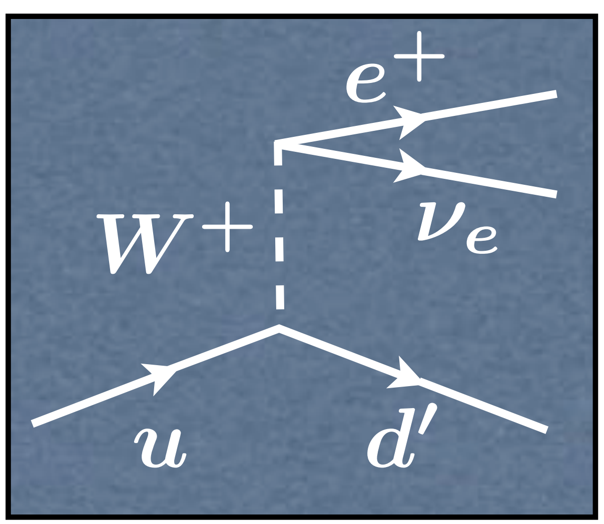
CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$



CKM Fitter



tree

$$K \to \pi \ell \nu \qquad B \to \pi \ell \nu$$

$$D \to \pi \ell \nu \qquad D \to K \ell \nu \qquad B_{(s)} \to D^{(*)}_{(s)} \ell \nu$$

$$B^0_{(s)} - \bar{B}^0_{(s)} \qquad B_c \to J/\psi\, \ell \nu$$

CKM matrix from Higgs couplings

LH SU(2) doublets $Q^i_L = \begin{pmatrix} u'^i \\ d'^i \end{pmatrix}_L$ RH SU(2) singlets $u^i_R \quad d^i_R$

Interact with gauge bosons in covariant derivative

$$\mathcal{L}_{\text{quark}} = \bar{Q}^i_L \, i\not{D} \, Q^i_L + \bar{u}^i_R \, i\not{D} \, u^i_R + \bar{d}^i_R \, i\not{D} \, d^i_R$$

Gives rise to weak current $J^{\mu,+}_{\text{weak}} = \bar{u}'^i_L \gamma^\mu d'^i_L$

The coupling to the Higgs field is not apparently diagonal in generation

$$\mathcal{L}_{\text{quark},\phi} = -\sqrt{2}\Big[\lambda^{ij}_d \bar{Q}^i_L \, \phi \, d^j_R + \lambda^{ij}_u \bar{Q}^i_{La} \, \epsilon_{ab} \phi^\dagger_b \, u^j_R + \text{h.c.}\Big]$$

Fields may be transformed to mass basis

$$\mathcal{L}_{\text{quark},\phi}\big|_{vev} = -\sum_i \left(m^i_d \bar{d}^i_L d^i_R + m^i_u \bar{u}^i_L u^i_R + \text{h.c.}\right)$$

Showing the weak current allows mixing between generations

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Role of Lattice QCD

Models

Experiment

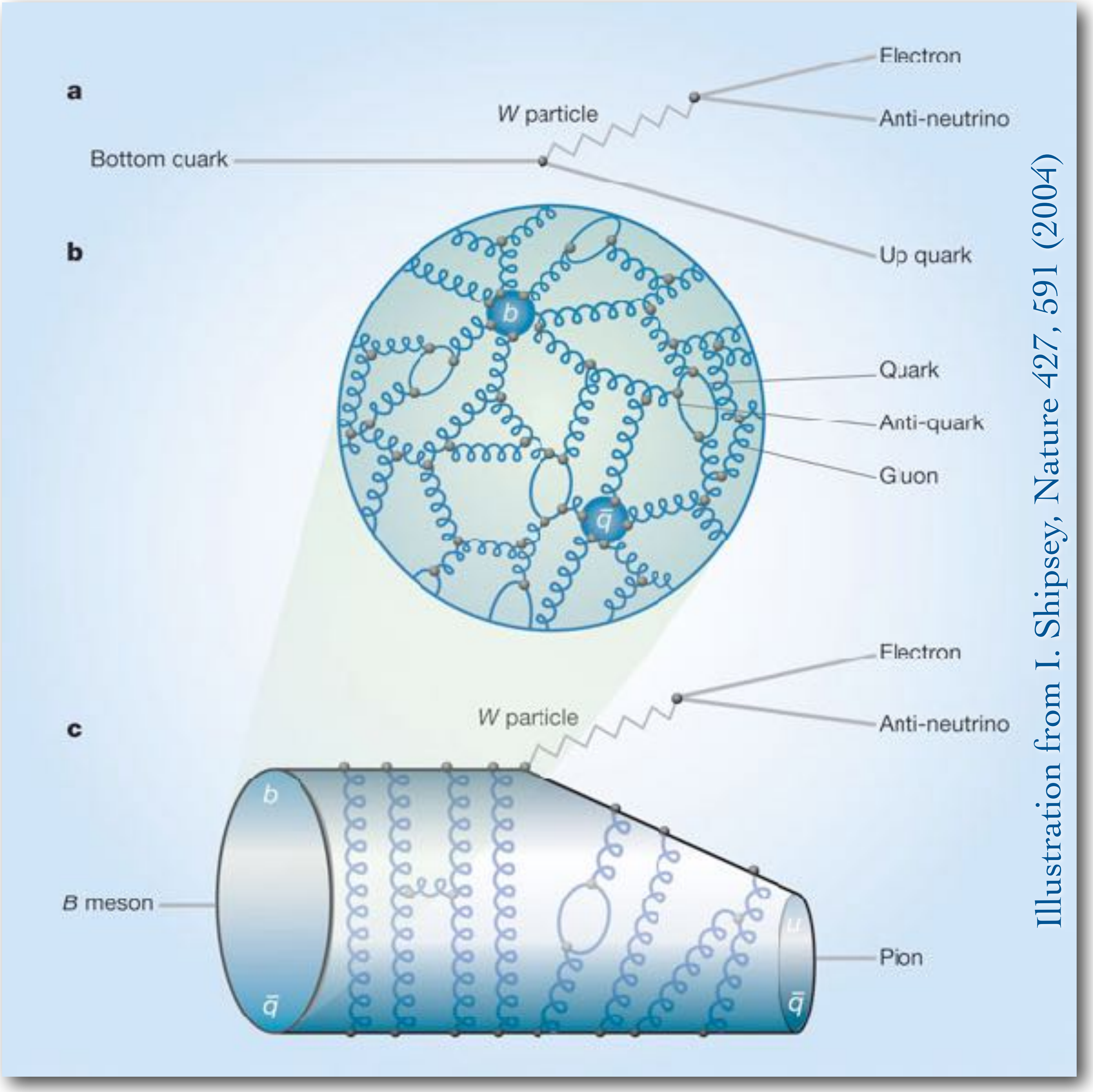
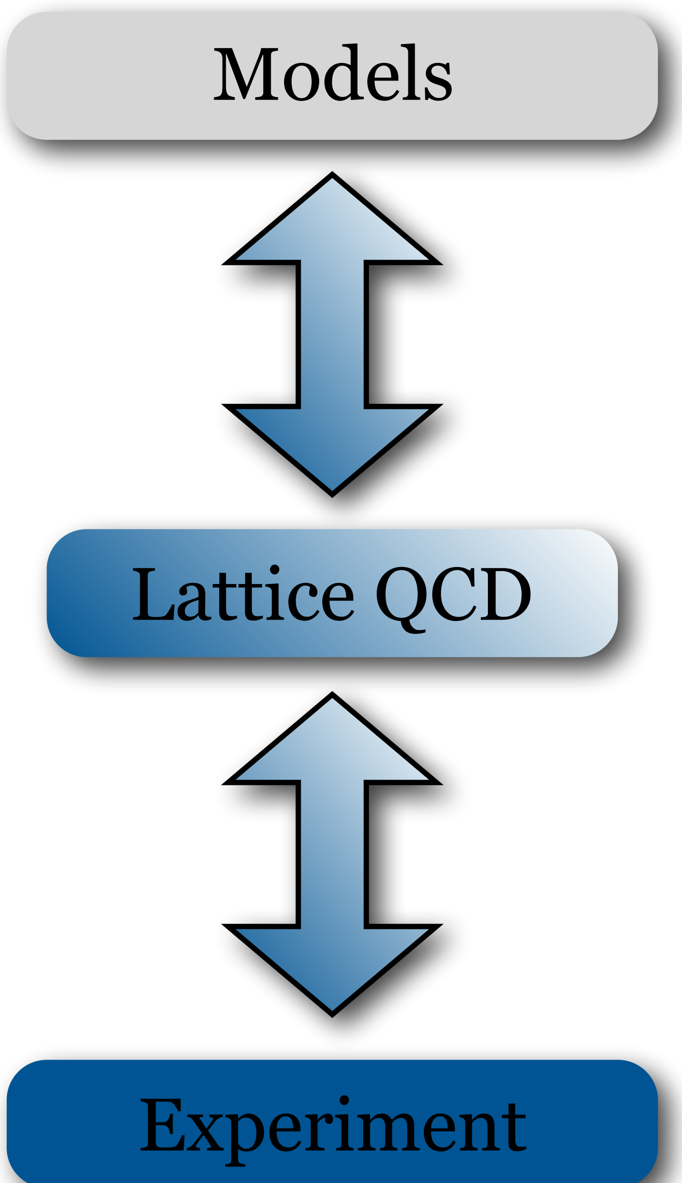


Illustration from I. Shipsey, Nature 427, 591 (2004)

Role of Lattice QCD



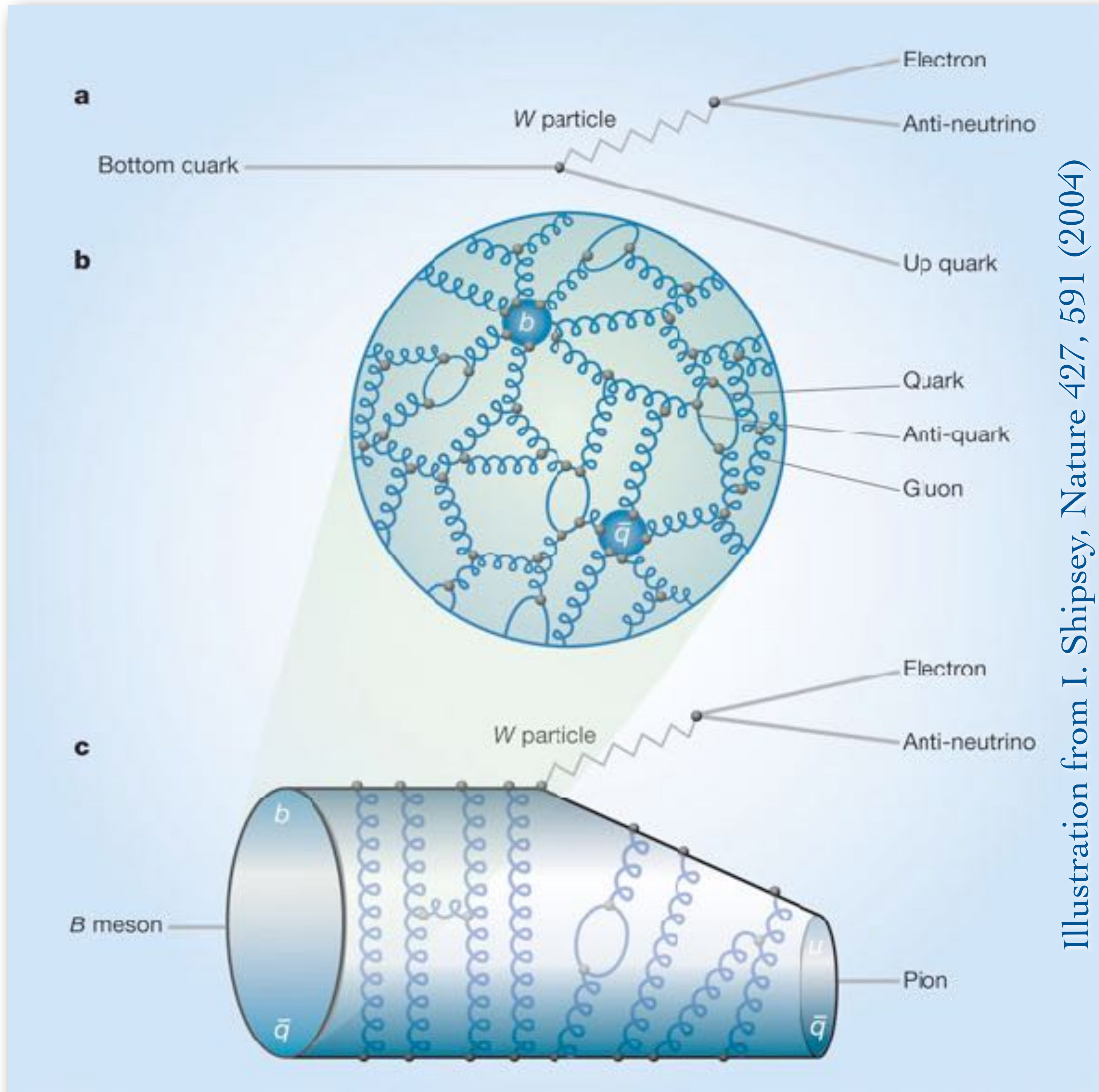


Illustration from I. Shipsey, Nature 427, 591 (2004)

Lattice QCD

- Use methods of effective field theory and renormalization to turn a ***quantum physics*** problem into a ***statistical physics*** problem

- ***Quarks*** propagating through strongly interacting QCD ***glue*** + ***sea*** of quark-antiquark bubbles

- Numerically evaluate path integrals using Monte Carlo methods: ***importance sampling*** & ***correlation functions***

- Numerical challenge: solving $M\,x = b$ where M is big and has a ***diverging condition number*** as $am_q \rightarrow 0$ (vanishing lattice spacing × light quark mass)

Lattice QCD in a nutshell

QFT : Imaginary-time path integral

$$\langle \boldsymbol{J}(\boldsymbol{z}')\boldsymbol{J}(\boldsymbol{z})\rangle = \frac{1}{\boldsymbol{Z}}\int [\boldsymbol{d}\psi][\boldsymbol{d}\bar{\psi}][\boldsymbol{d}\boldsymbol{U}]\, \boldsymbol{J}(\boldsymbol{z}')\boldsymbol{J}(\boldsymbol{z})\, \boldsymbol{e}^{-\boldsymbol{S}_{\boldsymbol{E}}}$$

SFT : Sum over all microstates

$$\langle \boldsymbol{J}(\boldsymbol{z}')\boldsymbol{J}(\boldsymbol{z})\rangle = \frac{1}{\boldsymbol{Z}}\,\mathbf{Tr}\,\left[\boldsymbol{J}(\boldsymbol{z}')\boldsymbol{J}(\boldsymbol{z})\, \boldsymbol{e}^{-\boldsymbol{\beta}\boldsymbol{H}}\right]$$

Use the same numerical methods!

Monte Carlo Calculation : Find and use field "configurations" which dominate the integral/sum

Lattice QCD in a nutshell

Gluonic expectation values

$$\begin{aligned}\langle \Theta \rangle &= \frac{1}{Z}\int [d\psi][d\bar{\psi}][dU]\, \Theta[U]\, e^{-S_g[U]-\bar{\psi}Q[U]\psi} \\ &= \frac{1}{Z}\int [dU]\, \Theta[U]\ \det Q[U]\, e^{-S_g[U]}\end{aligned}$$

Fermionic expectation values

$$\langle \bar{\psi}\Gamma\psi \rangle = \int [dU]\, \frac{\delta}{\delta\bar{\zeta}}\Gamma\frac{\delta}{\delta\zeta}\, e^{-\bar{\zeta}Q^{-1}[U]\zeta}\ \det Q[U] e^{-S_g[U]}\Big|_{\zeta,\bar{\zeta}\,\to\, 0}$$

Lattice QCD in a nutshell

Gluonic expectation values

$$\langle \Theta \rangle = \frac{1}{Z} \int [d\psi][d\bar{\psi}][dU]\, \Theta[U]\, e^{-S_g[U] - \bar{\psi} Q[U] \psi}$$

$$= \frac{1}{Z} \int [dU]\, \Theta[U]\, \det Q[U]\, e^{-S_g[U]}$$

Fermionic expectation values

Probability weight

$$\langle \bar{\psi} \Gamma \psi \rangle = \int [dU]\, \frac{\delta}{\delta \bar{\zeta}} \Gamma \frac{\delta}{\delta \zeta}\, e^{-\bar{\zeta} Q^{-1}[U] \zeta}\, \det Q[U] e^{-S_g[U]} \Big|_{\zeta, \bar{\zeta} \to 0}$$

Lattice QCD in a nutshell

Gluonic expectation values

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Probability weight

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Determinant in probability weight difficult

1) Requires nonlocal updating; 2) Matrix becomes singular

Lattice QCD in a nutshell

Gluonic expectation values

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$$= \frac{1}{Z} \int [dU]\, \Theta[U]\, \det Q[U]\, e^{-S_g[U]}$$

Fermionic expectation values

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Determinant in probability weight difficult

1) Requires nonlocal updating; 2) Matrix becomes singular

Partial quenching =

different mass for valence Q^{-1} than for sea $\det Q$

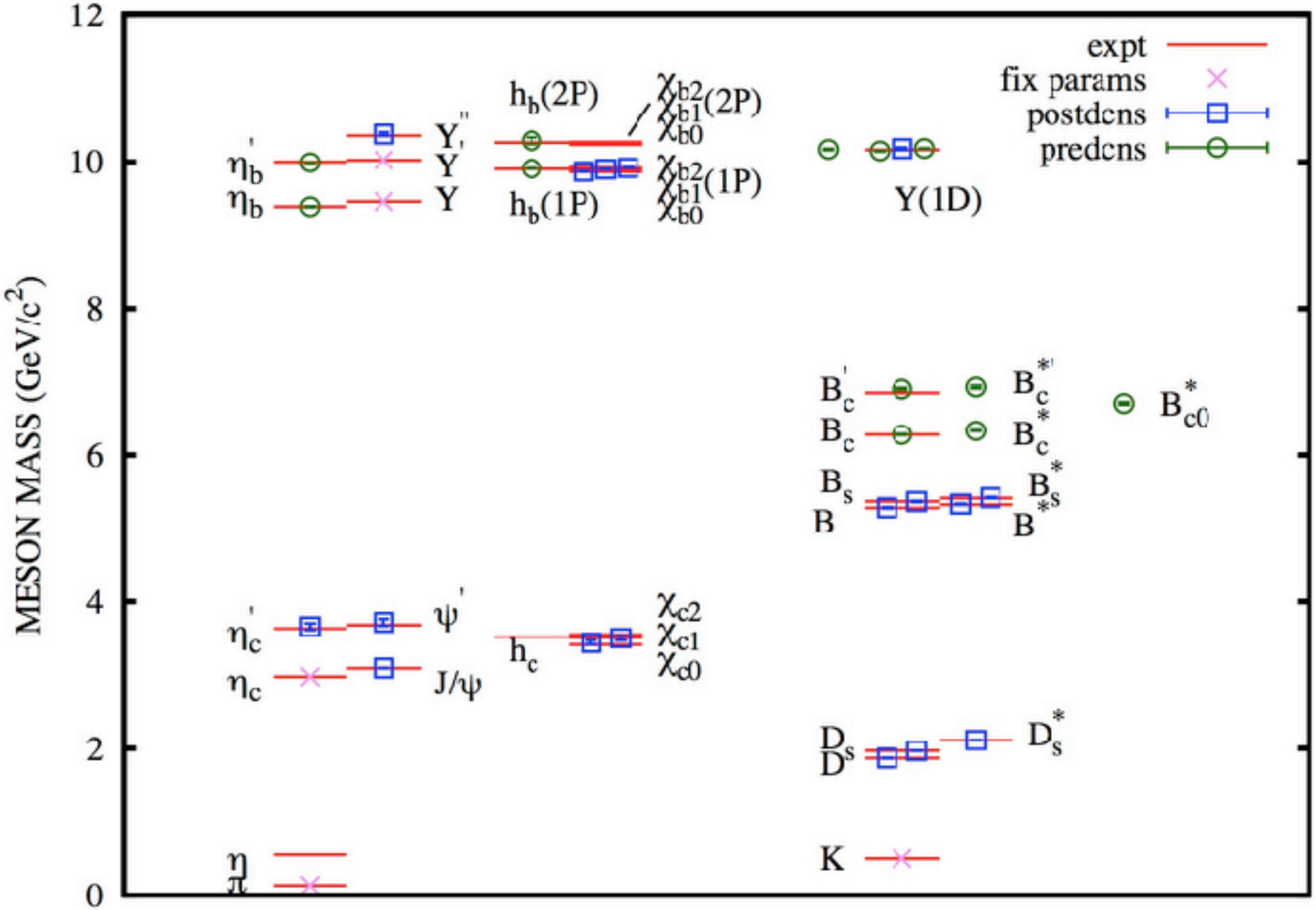
Lattice QCD

$$\langle \Phi_\pi(z) V_\mu(y) \Phi_B(x) \rangle = \frac{1}{Z} \int [d\psi][d\bar{\psi}][dU]\, \Phi_\pi(z) V_\mu(y) \Phi_B(x)\, e^{-S[\psi,\bar{\psi},U]}$$

- Imaginary time formulation: path integrands real, non-negative
- Discrete lattice points: regulates field theory
- Sharply peaked path integrand: permits importance sampling

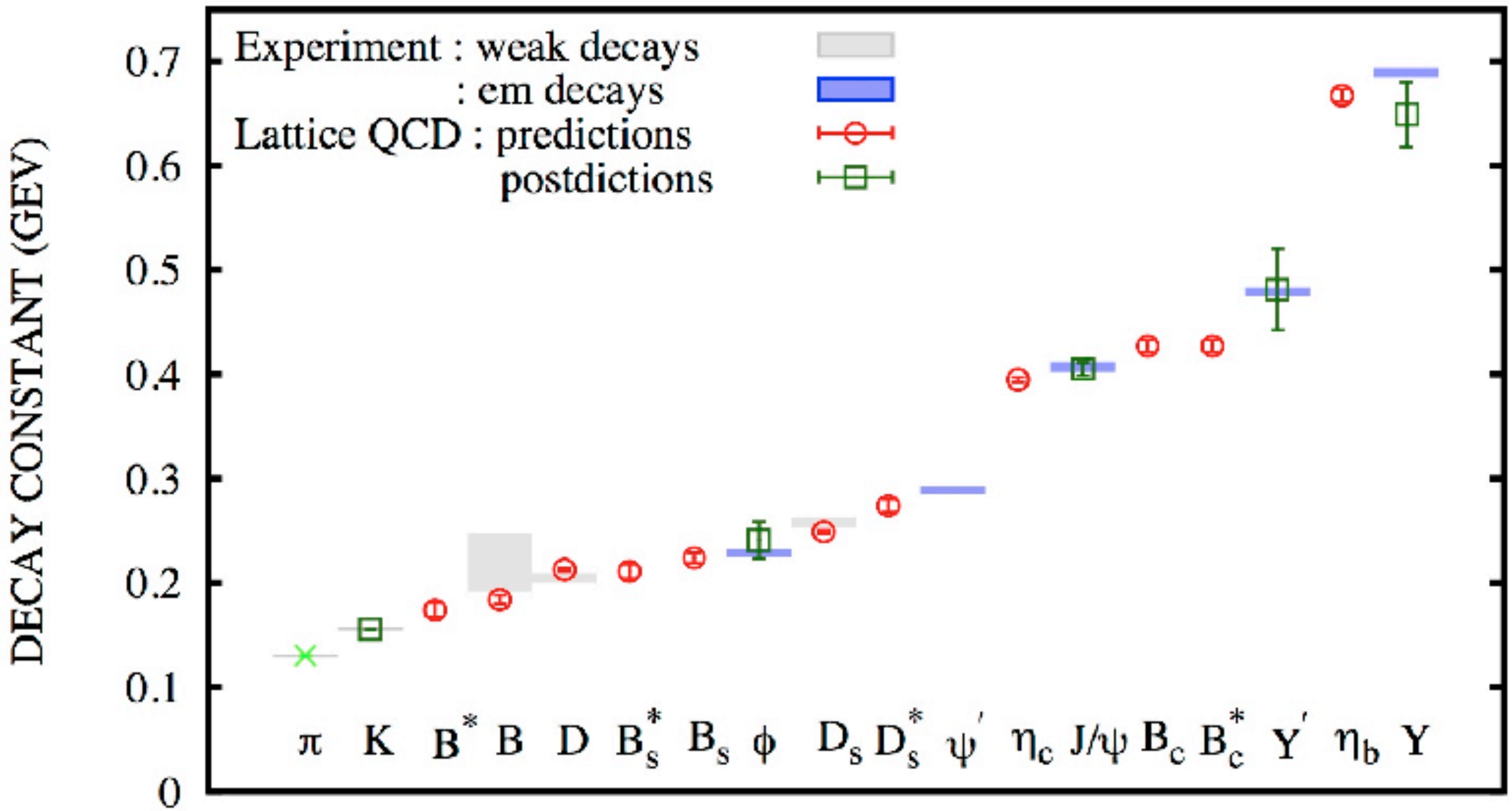
Systematic error	Controllable limit	Theory
Lattice volume	$L \gg 1/m_\pi$	Chiral pert. th. Brute force
Lattice spacing	$a \ll 1/\Lambda_{\rm QCD}$	Symanzik EFT
Light quark mass	$m_\pi \ll m_\rho, 4\pi f_\pi$	Chiral pert. th. Brute force
Heavy quark mass	$m_Q \gg 1/a$ $m_Q < 1/a$ $m_Q \approx 1/a$	NRQCD, HQET Extra-fine, extra-improvement Fermilab

Meson mass splittings



CTH Davies, [HPQCD Collaboration website]

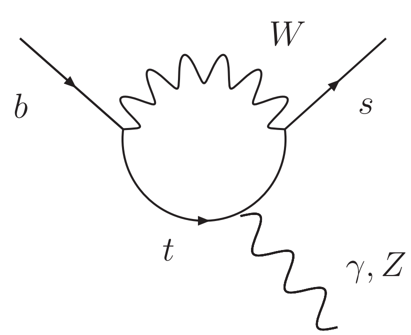
Decay constants



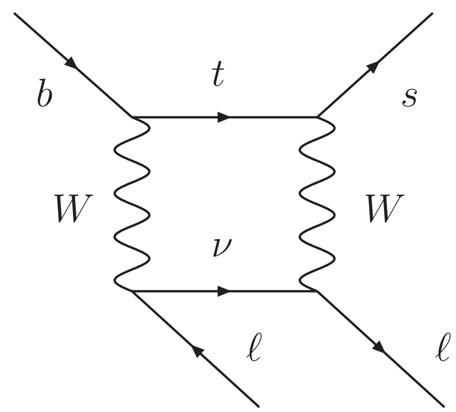
CTH Davies, [HPQCD Collaboration website]

Rare b decays

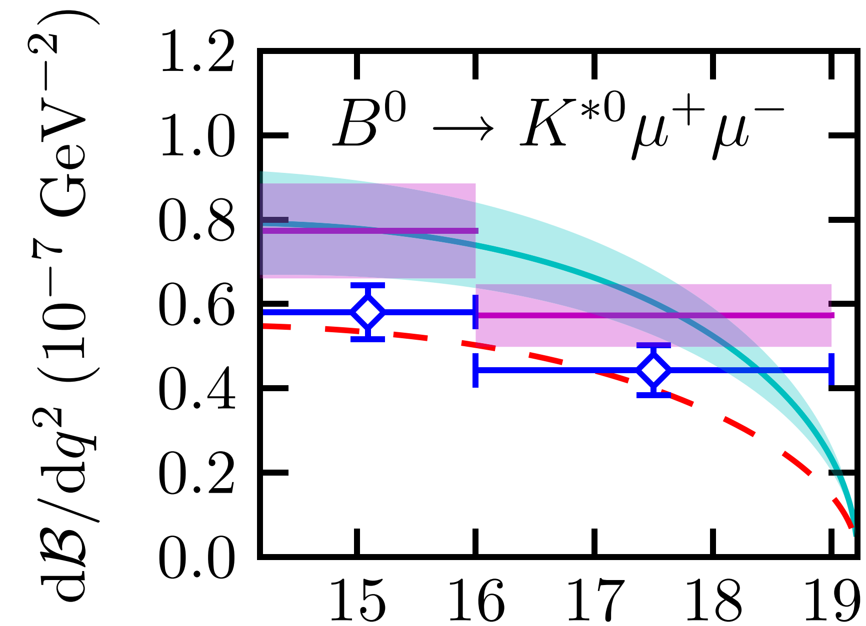
Flavour changing neutral decays

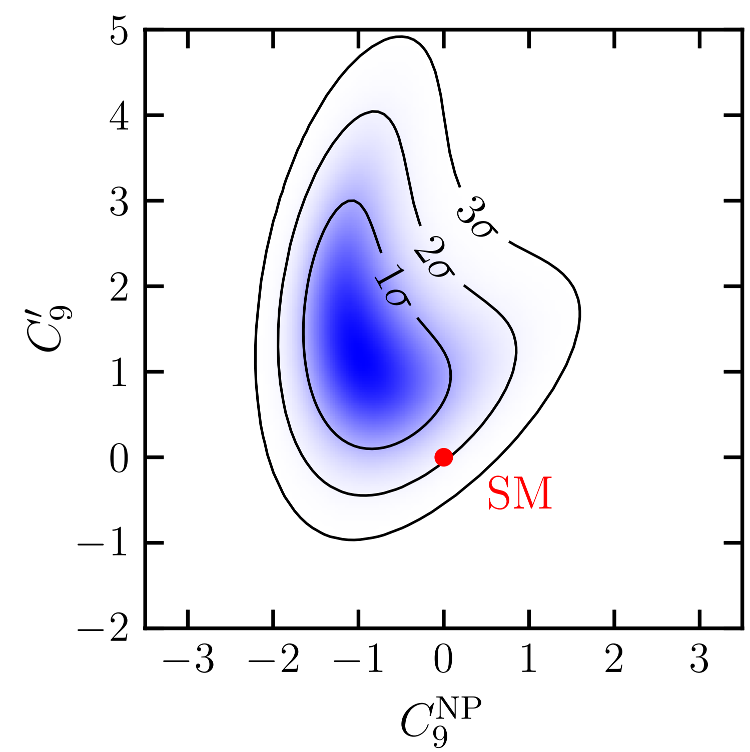


penguin



box





$B \to K^*\ell^+\ell^- \quad B_s \to \phi\ell^+\ell^-$

Horgan et al., (HPQCD) arXiv:1310.3722, arXiv:1310.3887

LQCD & DiRAC

UKQCD consortium

- 24 faculty at 8 UK institutions
- Membership/Leadership in several international collaborations (e.g. HPQCD, RBC-UKQCD, HadSpec, QCDSF, FastSum)
- Broad range of physics: quark flavour, hadron spectrum, hot/dense QCD; BSM theories of EWSB, dark matter
- Widespread impact: LHC, BES-III, Belle, JLab, J-PARC, FAIR, RHIC, NA62

Image credit: CIA World Factbook

DiRAC 2

- 2011: £15M BIS investment in national distributed HPC facility for particle & nuclear physics, cosmology, & theoretical astrophysics. Recurrent costs funded by STFC

- 2012: 5 systems deployed:
    - **Extreme scaling**:1.3 Pflop/s Blue Gene/Q (Edinburgh)
    - **Data Analytic**/**Data Centric**/**Complexity**: 3 tightly-coupled clusters with various levels of interconnectivity, memory, and fast I/O (Cambridge, Durham, Leicester)
    - **Shared Memory System** (**SMP**) (Cambridge)

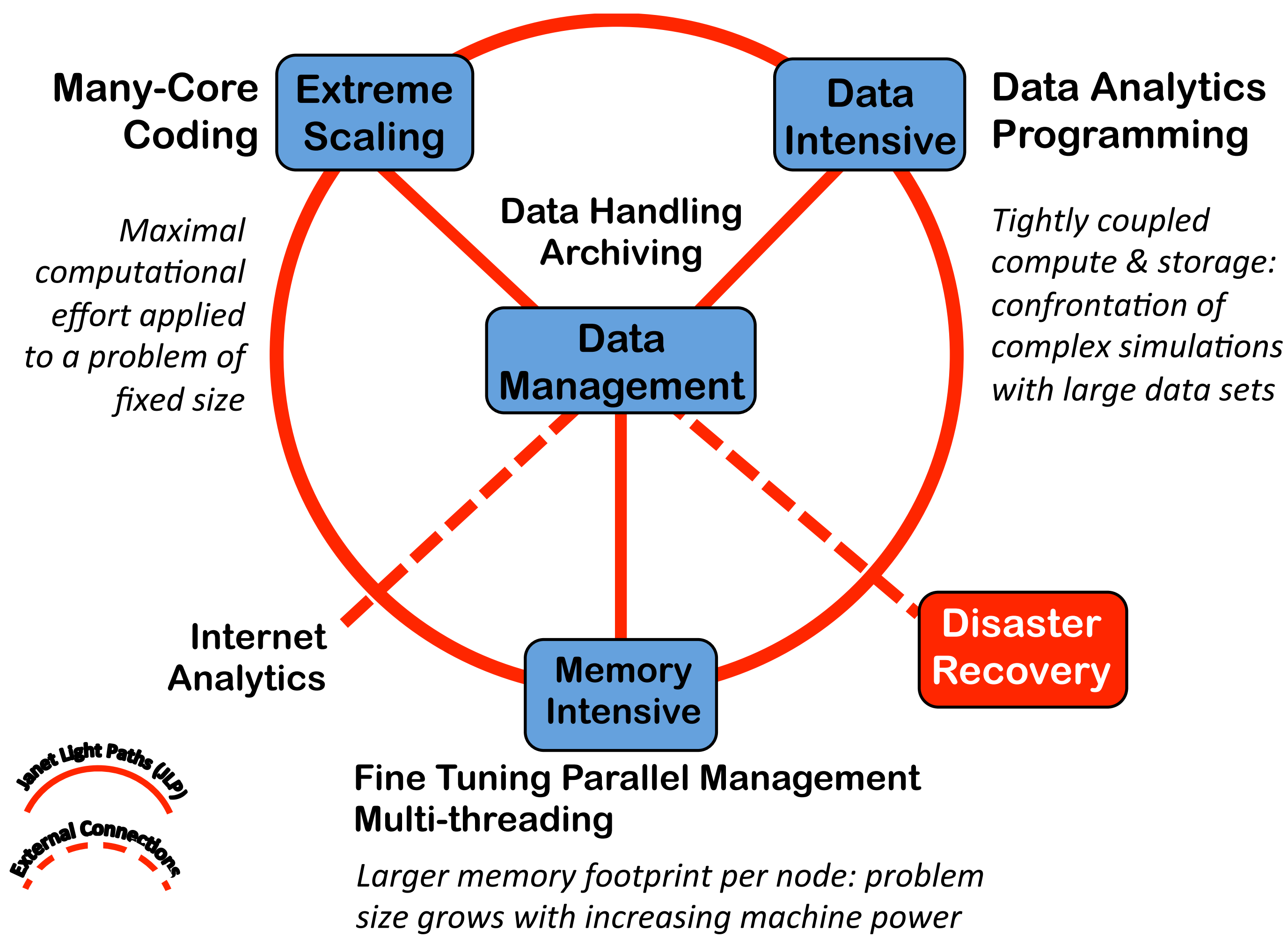
- Service started 1 December 2012

DiRAC 2 outputs

- 106 lattice publications, with 1977 citations (as of 20/7/2017)
- 765 publications in a broad scientific range (PPAN) — 35,365 citations (as of 20/7/2017)
- Gravitational waves, cosmology, galaxy & planet formation, exoplanets, MHD, particle pheno, nuclear physics
- Valuable resource for PDRA's & PhD students
- Scientific results, training in high performance computing

DiRAC 3

- Continued success requires continued investment
- Seek approx £25M capital investment to upgrade DiRAC-2 x10
- Running costs for staff and electricity
- Improve exploitation of research and HPC training impact with PDRA and PhD support (Big Data CDTs)
- Part of RCUK's <u>e-Infrastructure roadmap</u>



2011/12

DiRAC 2

Stop-gap funding:

2016/17 DiRAC 2.5

2017 DiRAC 2.5x

2018/19

DiRAC 3

DiRAC 2.5

After £1.67M capital injection

- **Extreme Scaling 2.5**: 1.3 Pflop/s Blue Gene/Q
- **Data Analytic 2.5**: Share of Peta5 system + continued access to Sandybridge system
  - Shared EPSRC/DiRAC/Cambridge: 25K Skylake cores + 1.0 Pflop/s GPU + 0.5 Pflop/s KNL service
- **Data Centric 2.5**: Over 14K cores, 128 GB RAM/node
- **Complexity 2.5**: 4.7K large-job cores + 3K small-job cores
- **SMP**: 14.8TB, 1.8K core shared memory service

DiRAC 2.5x

June 2017: £9M capital funding (BEIS), lifeline to DiRAC3:

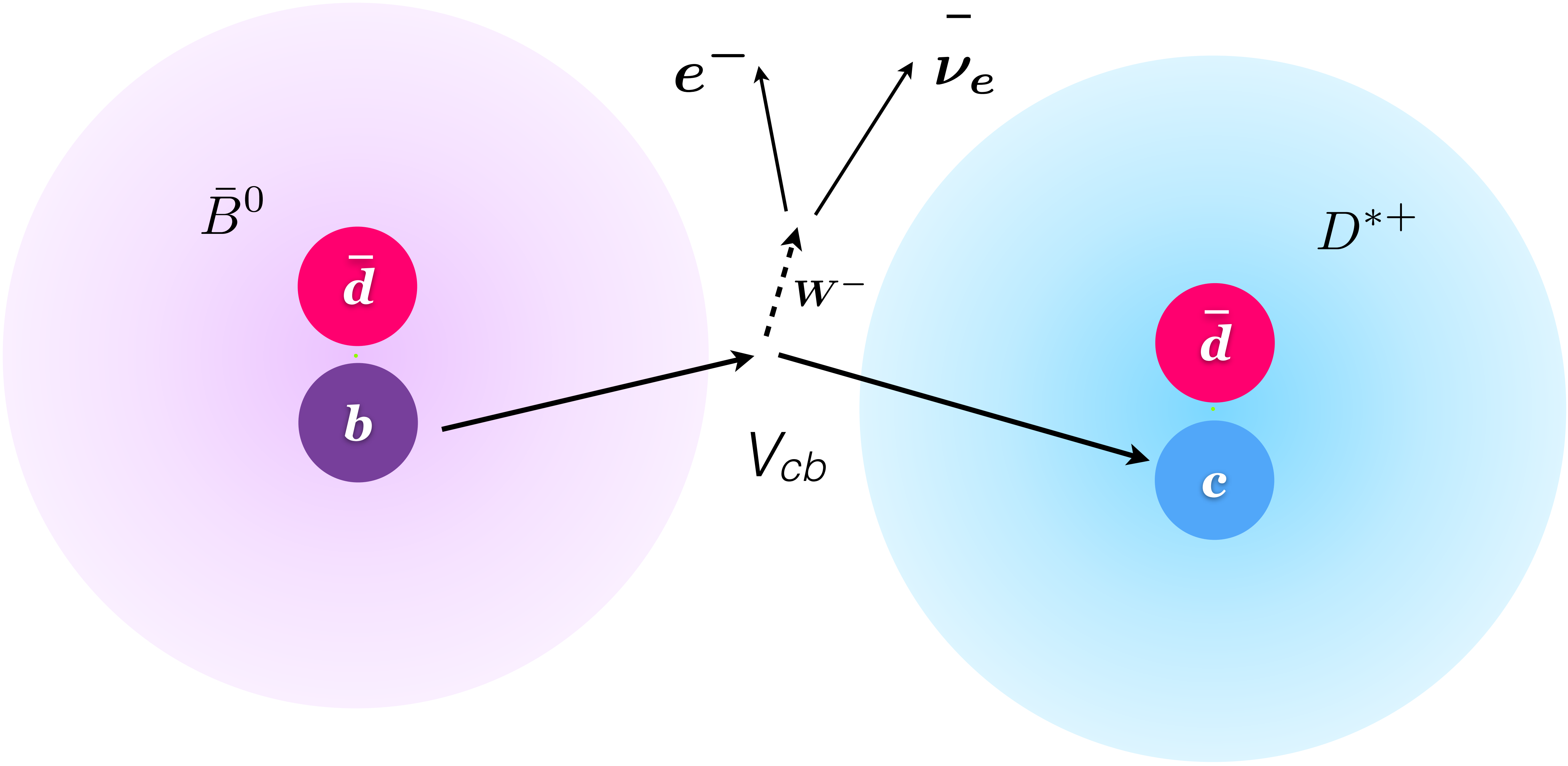
- Planned investment
  - **Extreme scaling**: 1024-node, 2.5 Pflop/s system
  - **Memory intensive**: 144 nodes, 4.6K cores, 110 TB RAM
  - **Data analytic**: 128 nodes, 4K cores, 256GB/node; hierarchy of fat nodes (1-6 TB); NVMe storage for data intensive workflows
- Additional storage at all DiRAC sites
- Procurement procedure: November 2017
- Target for hardware availability: April 2018

DiRAC & LQCD

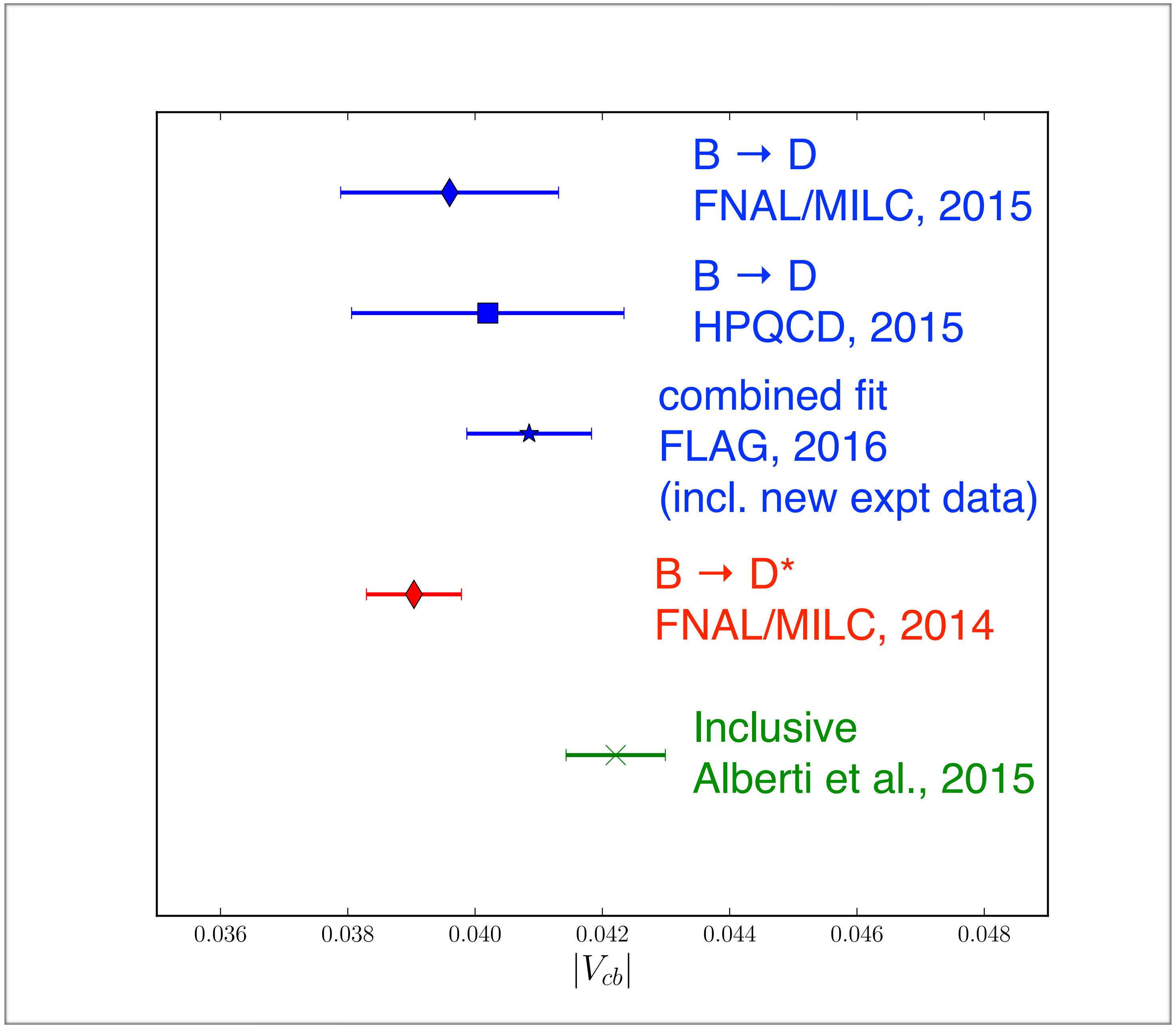
- Capital expenditure has come directly from BIS/BEIS, running costs through STFC
- DiRAC has allowed the UK to be a major contributor to world-wide Lattice QCD (and BSM) efforts
- High precision theory needed to make the most of high precision experiment

$B \to D^* l \nu$ and V_{cb}

B → D* l ν

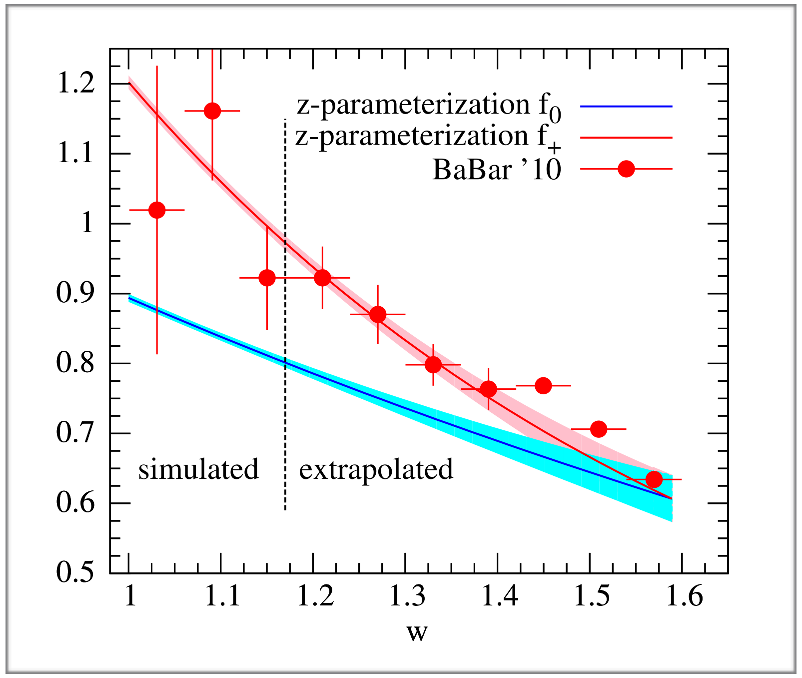


$|V_{cb}|$ (before 2/2017)



Published $B \to D$

Fermilab / MILC



Source	$f_+(\%)$	$f_0(\%)$
Statistics+matching+χPT cont. extrap.	1.2	1.1
(Statistics)	(0.7)	(0.7)
(Matching)	(0.7)	(0.7)
(χPT/cont. extrap.)	(0.6)	(0.5)
Heavy-quark discretization	0.4	0.4
Lattice scale r_1	0.2	0.2
Total error	1.2	1.1

Bailey et al. (FNAL/MILC), arXiv:1503.07237

HPQCD

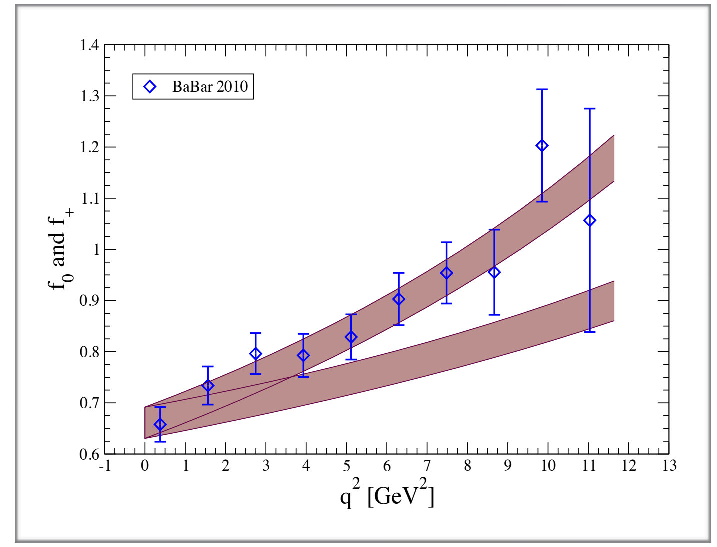


TABLE V. Error budget table for $|V_{cb}|$. The first three rows are from experiments, and the rest are from lattice simulations.

Type	Partial errors [%]
experimental statistics	1.55
experimental systematic	3.3
meson masses	0.01
lattice statistics	1.22
chiral extrapolation	1.14
discretization	2.59
kinematic	0.96
matching	2.11
electro-weak	0.48
finite size effect	0.1
total	5.34

Na et al. (HPQCD), arXiv:1505.03925

Published $B \to D^*$

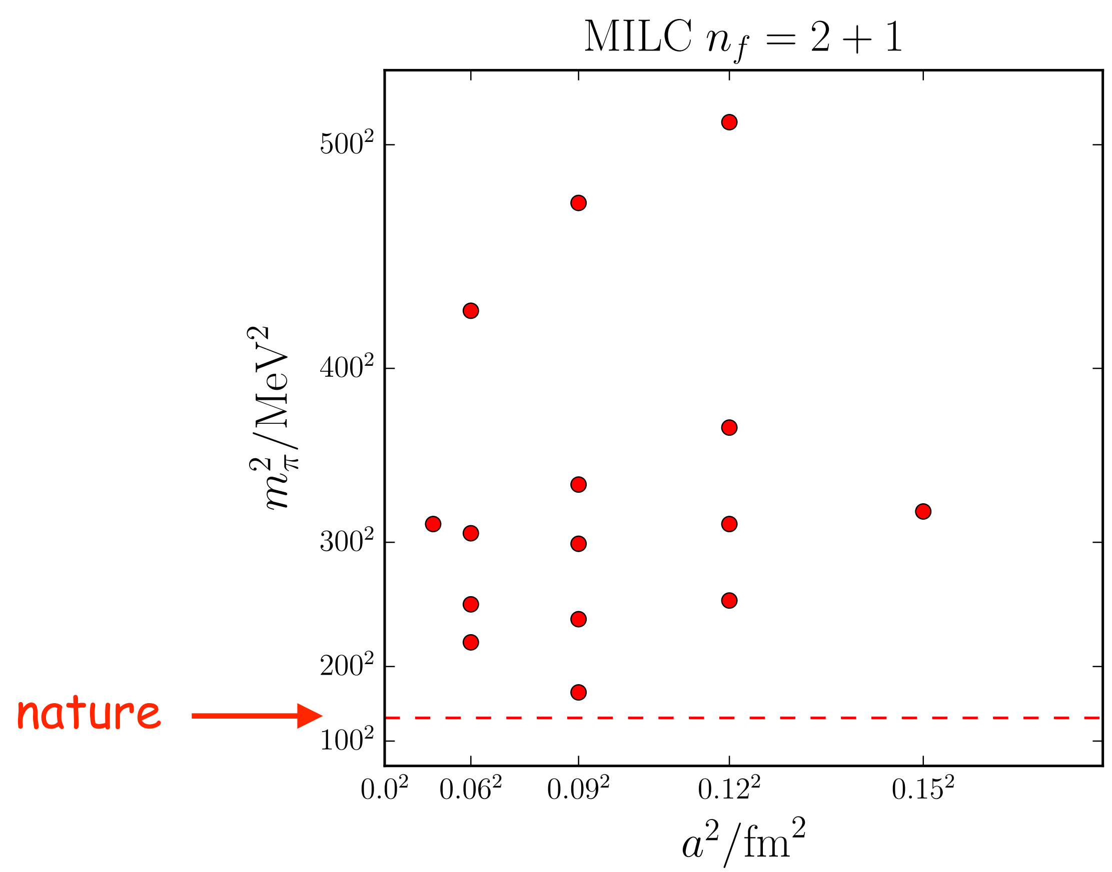
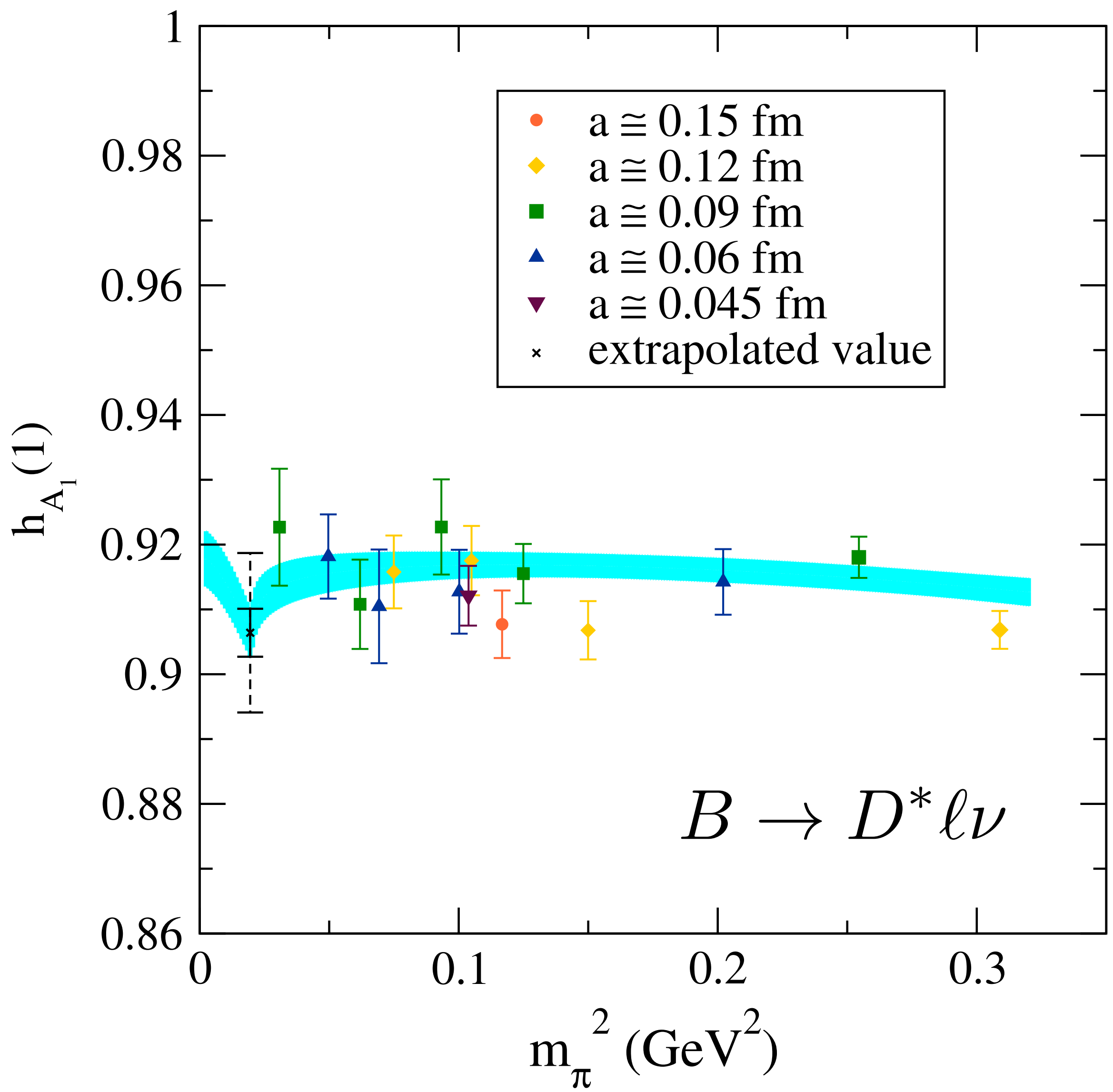


TABLE X. Final error budget for $h_{A_1}(1)$ where each error is discussed in the text. Systematic errors are added in quadrature and combined in quadrature with the statistical error to obtain the total error.

Uncertainty	$h_{A_1}(1)$
Statistics	0.4%
Scale (r_1) error	0.1%
χPT fits	0.5%
$g_{D^*D\pi}$	0.3%
Discretization errors	1.0%
Perturbation theory	0.4%
Isospin	0.1%
Total	1.4%

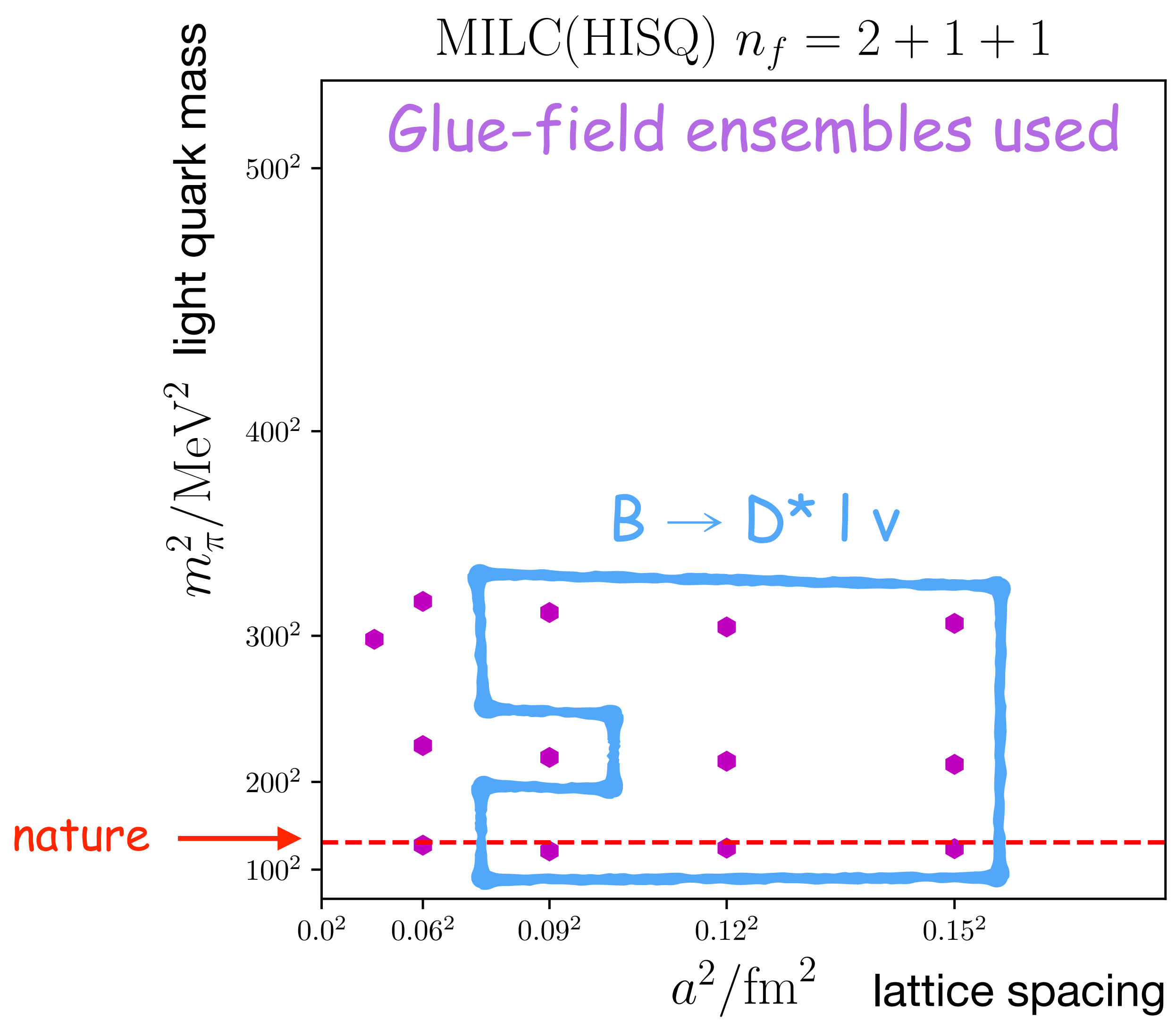
FNAL/MILC



Bailey et al. (FNAL/MILC), PRD89 (2014)

HPQCD calculation

Judd Harrison, Christine Davies, MBW (HPQCD), arXiv:1711.11013



- Statistically independent calculations from Fermilab/MILC
- HISQ vs. AsqTad light/strange
- HISQ vs. FNAL charm
- NRQCD vs. FNAL bottom

Zero recoil

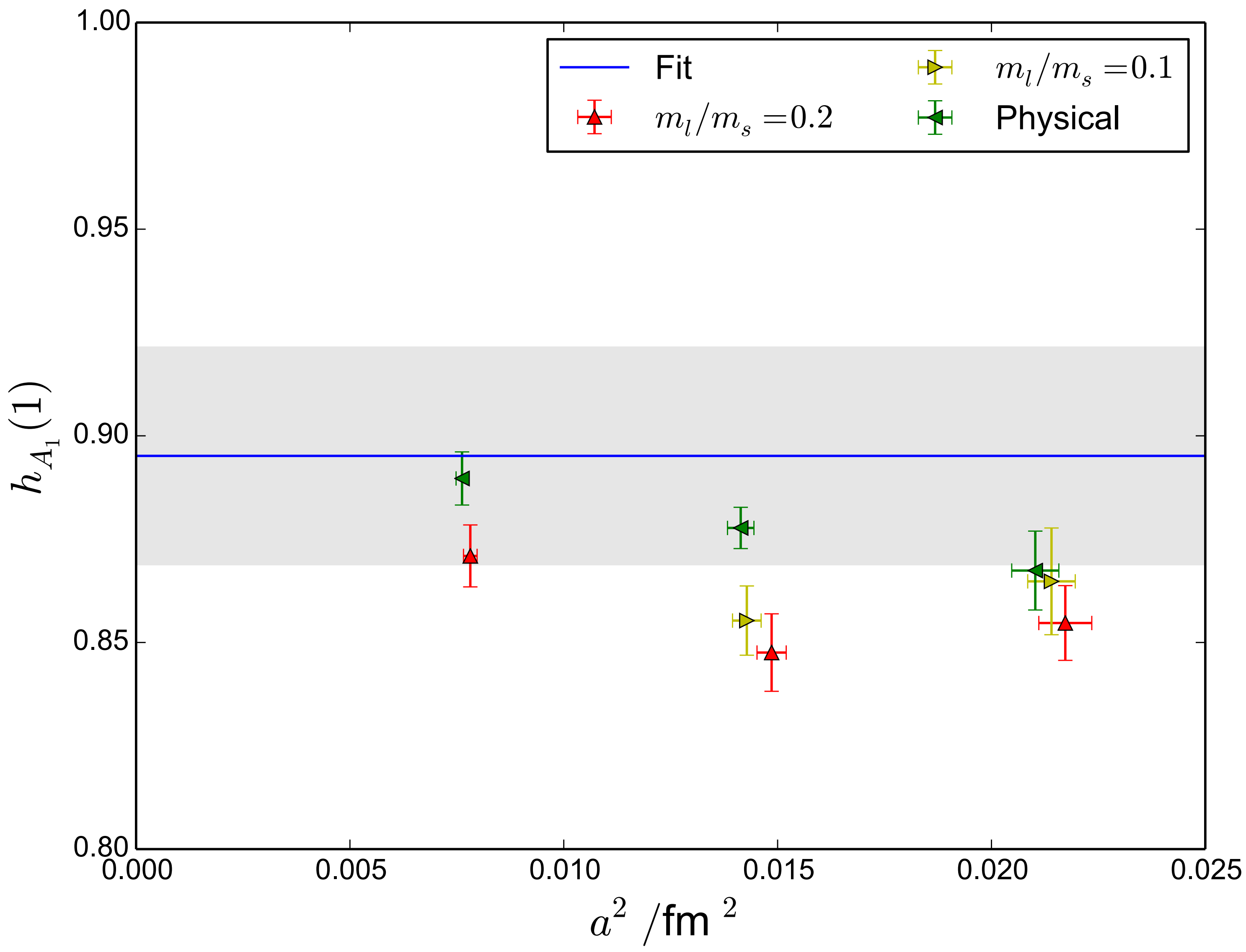
$$\frac{d\Gamma}{dw}(\bar{B}^0 \to D^{*+} l^- \bar{\nu}_l) = \frac{G_F^2 M_{D^*}^3 |\bar{\eta}_{EW} V_{cb}|^2}{4\pi^3}(M_B - M_{D^*})^2 \sqrt{w^2 - 1}\,\chi(w)|\mathcal{F}(w)|^2$$

$$\chi(1) = 1 \qquad\qquad \mathcal{F}(1) = h_{A_1}(1) = \frac{M_B + M_{D^*}}{2\sqrt{M_B M_{D^*}}} A_1(q^2_{\mathrm{max}})$$

$$\langle D^*(p', \epsilon)|\bar{q}\gamma^\mu\gamma^5 Q|B(p)\rangle = 2M_{D^*} A_0(q^2)\frac{\epsilon^* \cdot q}{q^2} q^\mu + (M_B + M_{D^*})A_1(q^2)\left[\epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^\mu\right]$$
$$- A_2(q^2)\frac{\epsilon^* \cdot q}{M_B + M_{D^*}}\left[p^\mu + p'^\mu - \frac{M_B^2 - M_{D^*}^2}{q^2} q^\mu\right].$$

$$\langle D^*(p', \epsilon)|\bar{q}\gamma^\mu Q|B(p)\rangle = \frac{2iV(q^2)}{M_B + M_{D^*}}\,\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p'_\rho p_\sigma$$

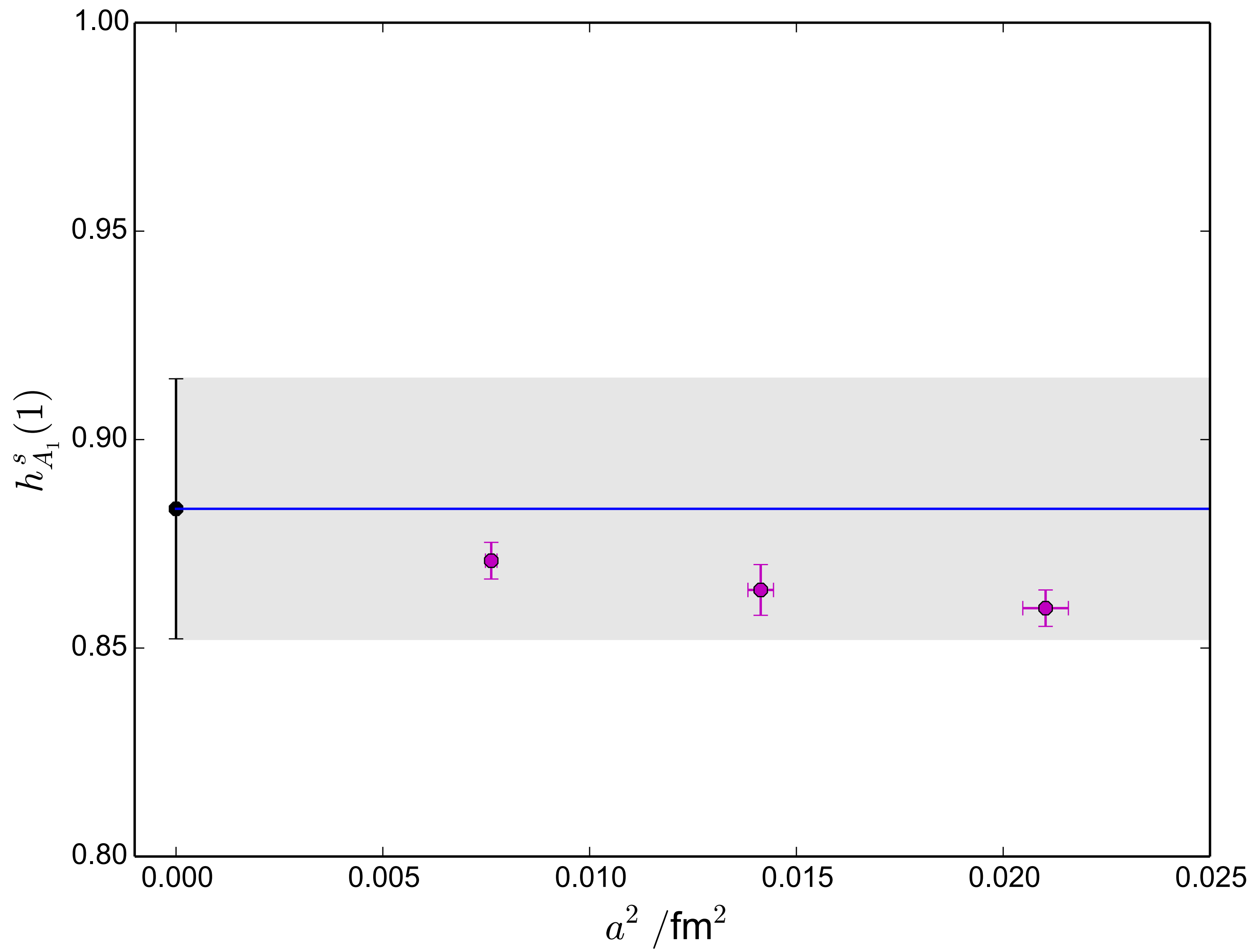
$B \to D^*$ — lattice spacing



Continuum-physical mass curve

$$\mathcal{F}^{B\to D^*}(1) = h_{A_1}(1) = 0.895(10)_{\text{stat}}(24)_{\text{sys}}$$

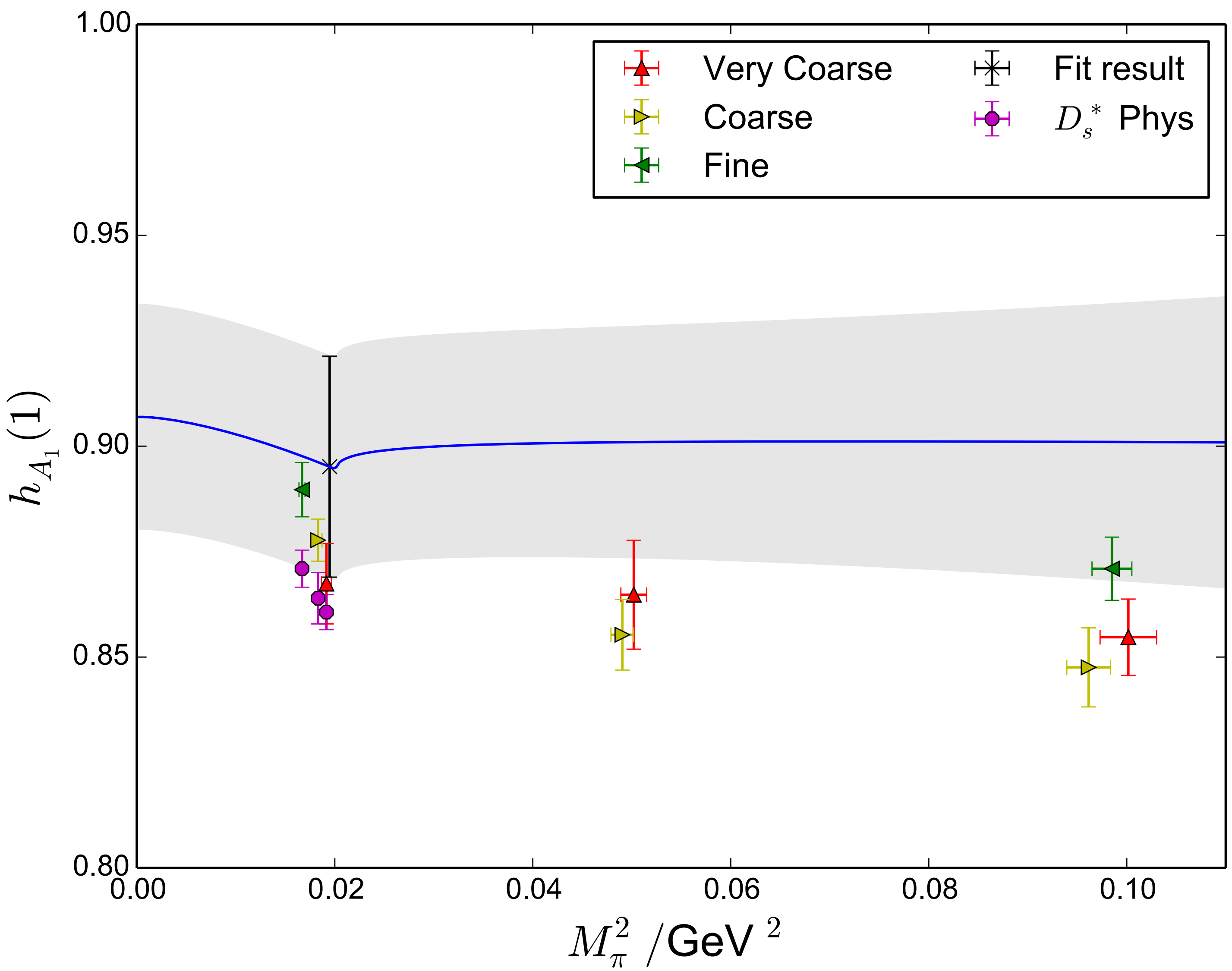
$B_s \to D_s^*$ — lattice spacing



Continuum-physical mass curve

$$\mathcal{F}^{B_s \to D_s^*}(1) = h^s_{A_1}(1) = 0.883(12)_{\rm stat}(28)_{\rm sys}$$

$B_{(s)} \rightarrow D_{(s)}{}^*$ — quark mass



Continuum-
physical mass
curve

Lattice results

$$\mathcal{F}^{B\to D^*}(1) = h_{A_1}(1) = 0.895(10)_{\rm stat}(24)_{\rm sys}$$

$$\mathcal{F}^{B_s\to D_s^*}(1) = h^s_{A_1}(1) = 0.883(12)_{\rm stat}(28)_{\rm sys}$$

$$\frac{\mathcal{F}^{B\to D^*}(1)}{\mathcal{F}^{B_s\to D_s^*}(1)} = \frac{h_{A_1}(1)}{h^s_{A_1}(1)} = 1.013(14)_{\rm stat}(17)_{\rm sys}$$

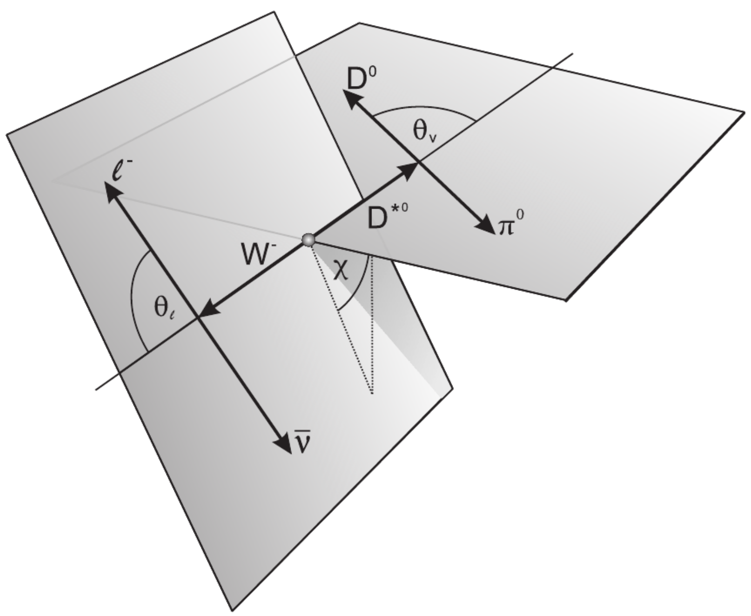
Uncertainty	$h_{A_1}(1)$	$h^s_{A_1}(1)$	$h_{A_1}(1)/h^s_{A_1}(1)$
α_s^2	2.1	2.5	0.4
$\alpha_s\Lambda_{\rm QCD}/m_b$	0.9	0.9	0.0
$(\Lambda_{\rm QCD}/m_b)^2$	0.8	0.8	0.0
a^2	0.7	1.4	1.4
$g_{D^*D\pi}$	0.2	0.03	0.2
Total systematic	2.7	3.2	1.7
Data	1.1	1.4	1.4
Total	2.9	3.5	2.2

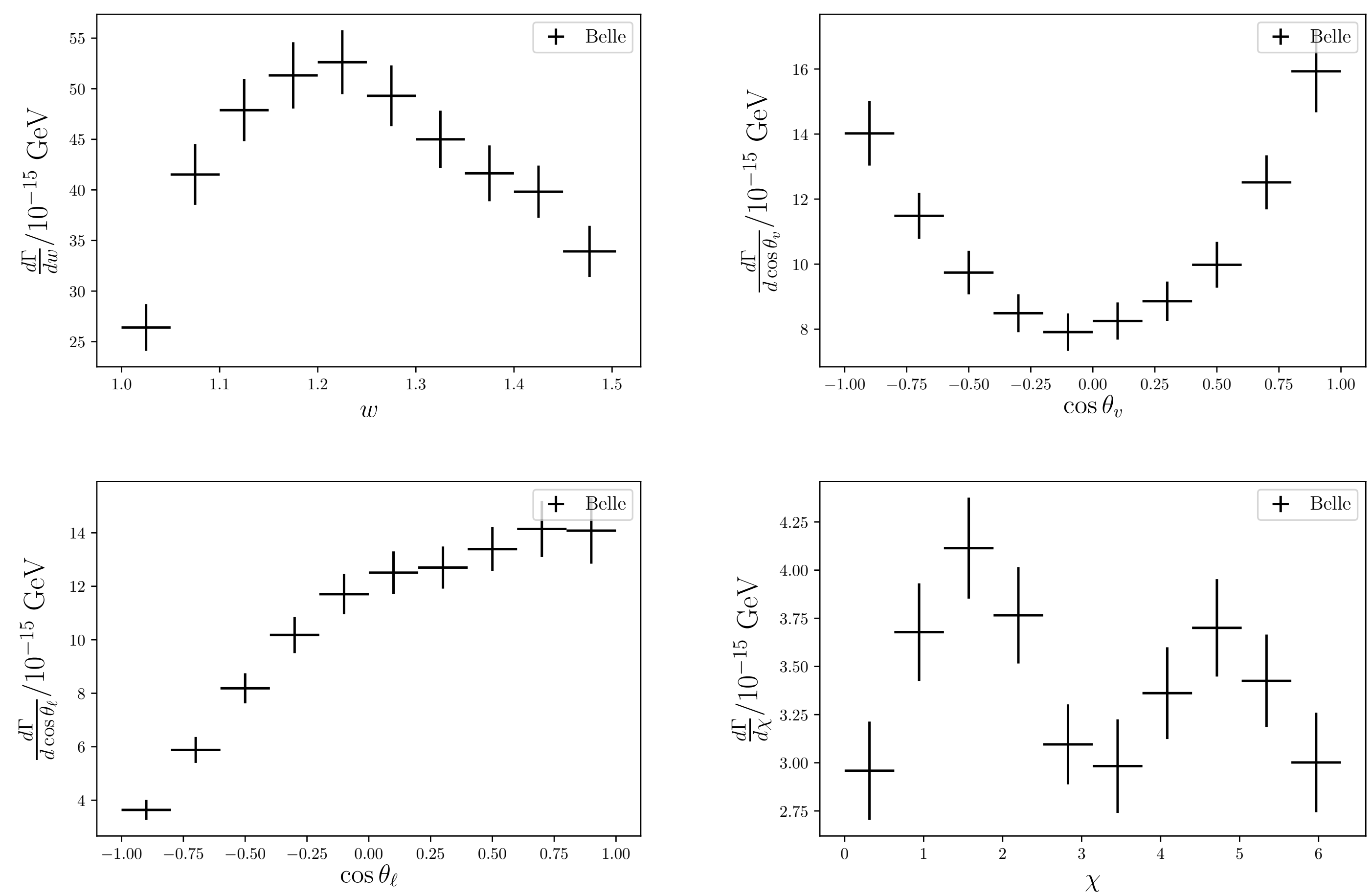
- Good agreement with Fermilab/MILC result $h_{A1}(1) = 0.906(4)(12)$
- Independent lattices
- Different heavy quark formulations

Implications for V_{cb}

unfolded Belle data

$B^- \to D^*(\to D\pi)\ell^-\bar{\nu}$





Abdesselam et al., arXiv:1702.01521

CLN parametrization

Form factors entering helicity amplitudes (massless leptons)

$$h_{A_1}(w) = h_{A_1}(1)[1 - 8\rho^2 z + (r_{h2r}\rho^2 + r_{h2})z^2 + (r_{h3r}\rho^2 + r_{h3})z^3]$$
$$R_1(w) = R_1(1) + r_{11}(w-1) + r_{12}(w-1)^2$$
$$R_2(w) = R_2(1) + r_{21}(w-1) + r_{22}(w-1)^2$$

$$w = v \cdot v'$$

Fixed:

$$r_{h2r} = 53\,, r_{h2} = -15\,, r_{h3r} = -231\,, r_{h3} = 91$$
$$r_{11} = -0.12\,, r_{12} = 0.05\,, r_{21} = 0.11\,, r_{22} = -0.06$$

Using this “tight” CLN parametrization

$$I = |\bar{\eta}_{EW} V_{cb}| h_{A_1}(1)$$

$$I_{\text{Belle}} = 0.0348(12)$$ (unfolded)

$$I_{\text{HFLAV}} = 0.03561(11)(44)$$

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CLN uncertainties

$$h_{A_1}(w) = h_{A_1}(1)[1 - 8\rho^2 z + (r_{h2r}\rho^2 + r_{h2})z^2 + (r_{h3r}\rho^2 + r_{h3})z^3]$$
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$$R_2(w) = R_2(1) + r_{21}(w-1) + r_{22}(w-1)^2$$

Coefficients calculated through *Λ/m* using HQET & sum rules

$$r_{h2r} = 53\,, r_{h2} = -15\,, r_{h3r} = -231\,, r_{h3} = 91$$ **BIG!**

$$r_{11} = -0.12\,, r_{12} = 0.05\,, r_{21} = 0.11\,, r_{22} = -0.06$$ small!

Ratios $V(q^2) = \frac{R_1(w)}{r'} h_{A_1}(w)$ $A_2(q^2) = \frac{R_2(w)}{r'} h_{A_1}(w)$

What are the uncertainties for the *r* 's? 20%? 100%?

See papers by Bigi, Gambino, Schacht; Grinstein & Kobach; Bernlochner et al.; Jaiswal, et al.

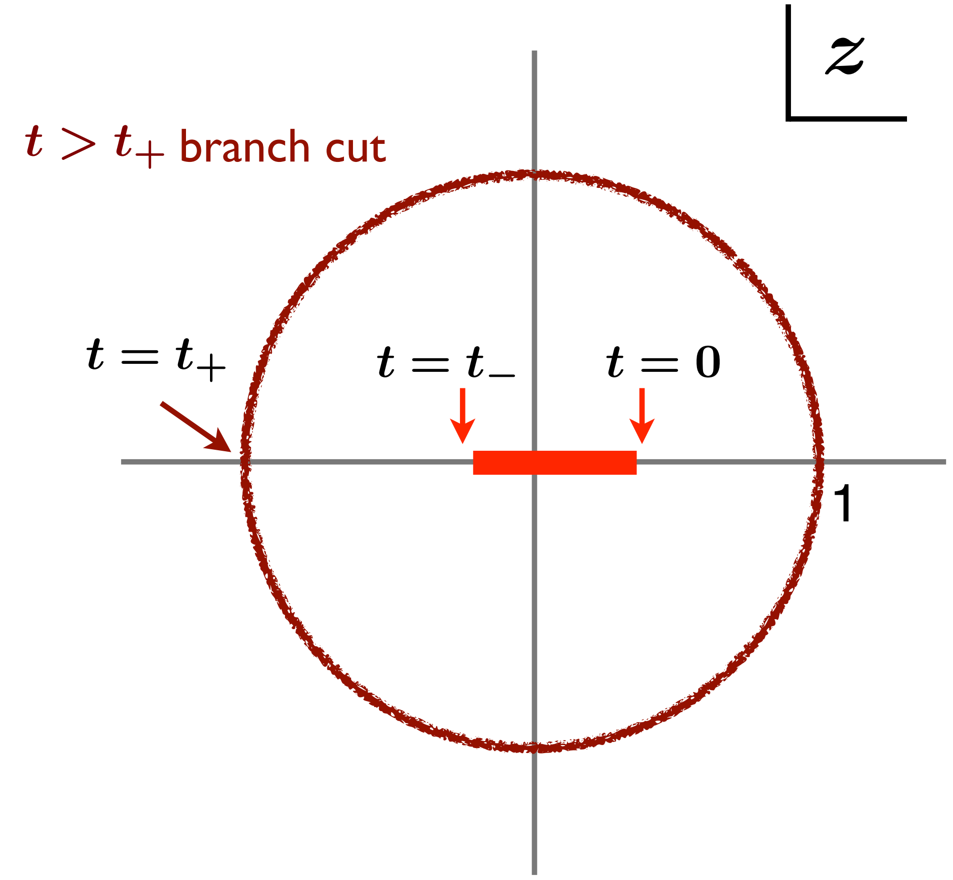
z-expansion

Series (z) expansion

$$t = q^2 \qquad t_\pm = (m_B \pm m_F)^2$$

Choose, *e.g.* $t_0 = t_-$

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$



Simplified series expansion

$$F(t) = \frac{1}{1 - t/m_{\mathrm{res}}^2} \sum_n a_n z^n$$

BGL parametrization

$$F(t) = Q_F(t) \sum_{k=0}^{K_F-1} a_k^{(F)} z^k(t,t_0) \qquad Q_F(t) = \frac{1}{B_n(z)\phi_F(z)}$$

Blaschke factor

$$B_n(z) = \prod_{i=1}^{n} \frac{z - z_{P_i}}{1 - z z_{P_i}} \qquad z_{P_i} = z(M_{P_i}^2, t_-)$$

Unitarity bounds

$$S_{fF} = \sum_{k=0}^{K_f-1} [(a_k^{(f)})^2 + (a_k^{(F_1)})^2] \leq 1 \qquad S_g = \sum_{k=0}^{K_g-1} (a_k^{(g)})^2 \leq 1$$

Predictions for B_c vector & axial vector resonances

$$M_B + M_{D^*} = 7.290 \text{ GeV}$$

M_{1^-}/GeV	method	Ref.	M_{1^+}/GeV	method	Ref.
6.335(6)	lattice	[77]	6.745(14)	lattice	[77]
6.926(19)	lattice	[77]	6.75	model	[79, 80]
7.02	model	[79]	7.15	model	[79, 80]
7.28	model	[81]	7.15	model	[79, 80]

BCL parametrization

Simple form which uses less theoretical information.

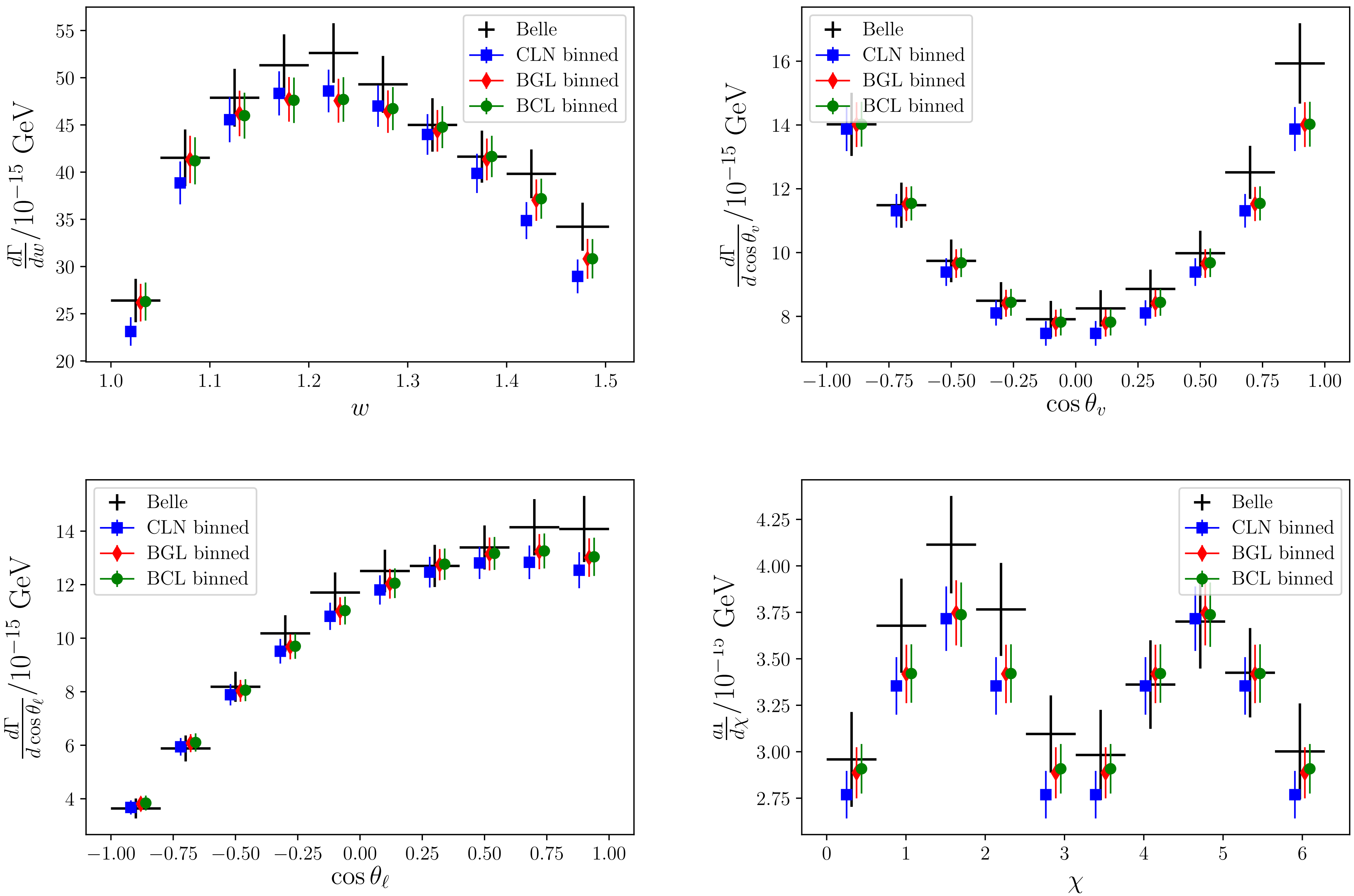
$$F(t) = Q_F(t) \sum_{k=0}^{K_F-1} a_k^{(F)} z^k(t, t_0) \qquad Q_F(t) = \frac{N_F}{1 - \frac{t}{M_P^2}}$$

Using BGL as a guide, choose N_f = 300, N_{F1} = 7000, N_g = 5

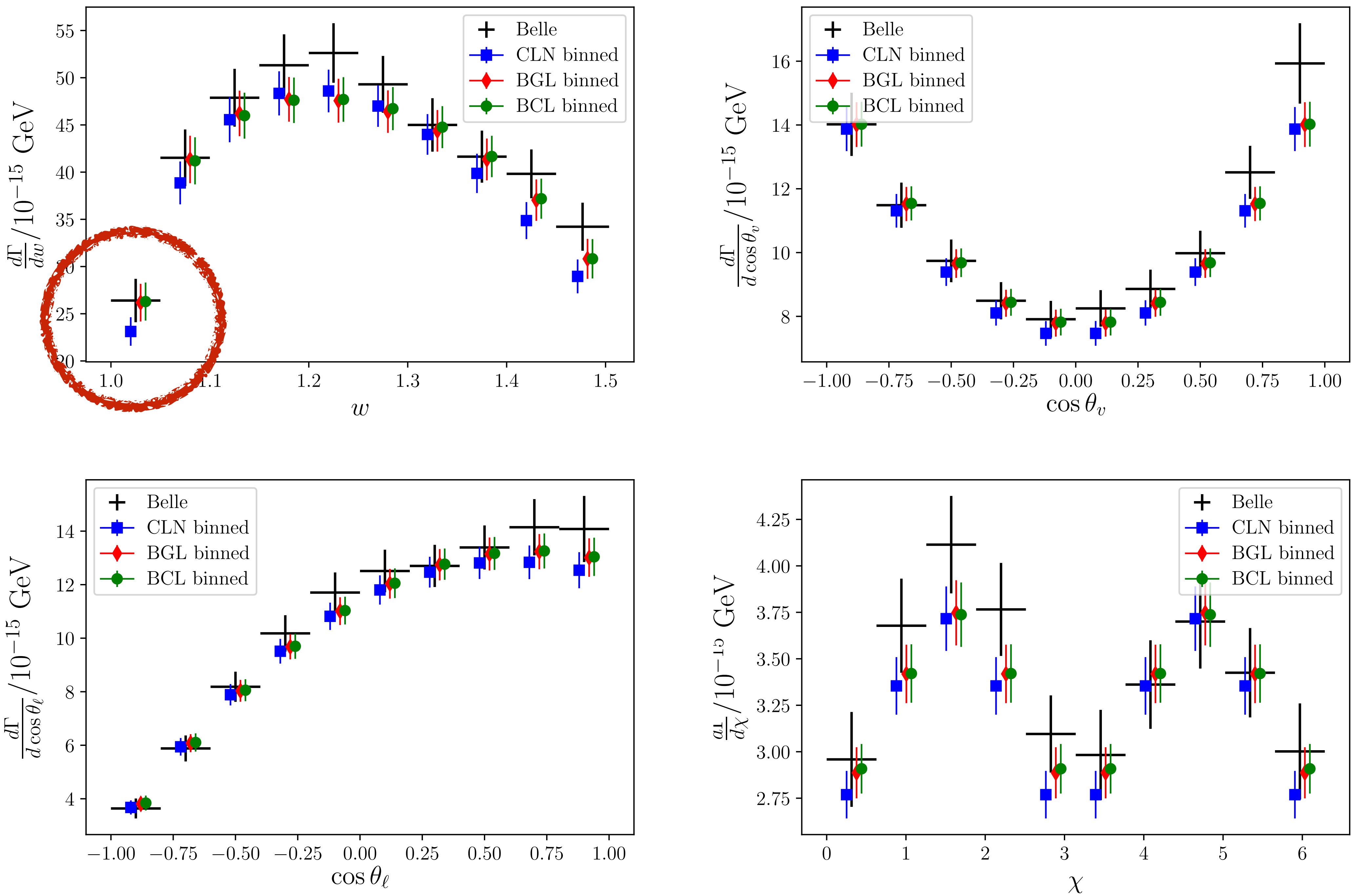
Clean baseline, against which affects of theoretical input (HQET, unitarity bounds) can be measured

fit	n_B^+	n_B^-	K	I	$a_0^{(f)}$	$a_1^{(f)}$	$a_0^{(F_1)}$	$a_1^{(F_1)}$	$a_0^{(g)}$	$a_1^{(g)}$	S_{fF}	S_g
BCL	–	–	2	0.0367(15)	0.01496(19)	−0.047(27)	0.002935(37)	−0.0029(27)	0.027(13)	0.77(44)	0.0025(26)	0.60(69)
BCL	–	–	3	0.0378(17)	0.01496(19)	−0.065(40)	0.002935(37)	−0.0135(82)	0.026(13)	0.82(46)	0.08(38)	0.67(75)
BCL	–	–	4	0.0382(18)	0.01497(19)	−0.310(42)	0.002936(37)	−0.0151(83)	0.109(16)	−0.29(37)	0.143(67)	0.10(22)
BCL	–	–	5	0.0382(18)	0.01497(19)	−0.310(42)	0.002936(37)	−0.0151(83)	0.109(16)	−0.29(37)	0.143(67)	0.10(22)

Fits to Belle data

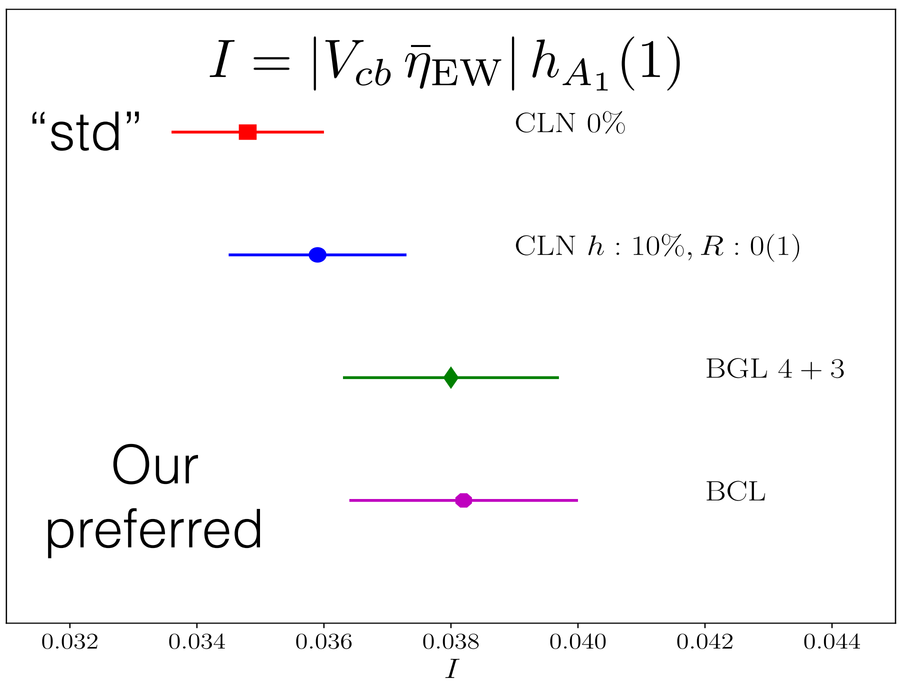


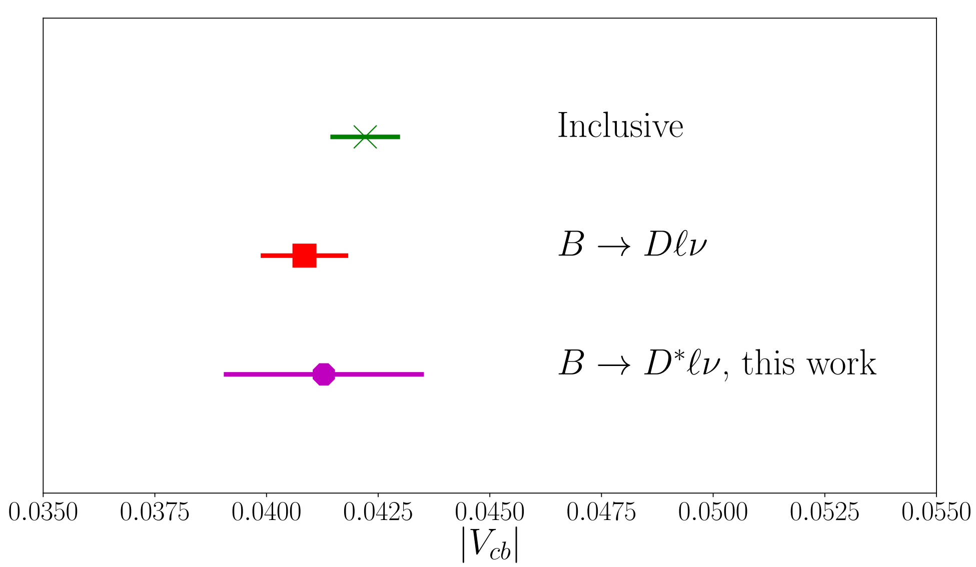
Fits to Belle data



Implications for V_{cb}

Different fit Ansätze





- Removal of theory assumptions resolves inclusive/exclusive tension, at least in Belle data
- Look forward to BaBar analysis
- Look forward to LQCD results at non-zero recoil

Conclusions

- Lattice field theory: nonperturbative, numerical approach connecting hadronic observables and fundamental quark interactions
- Lattice QCD plays an important role in studies of quark flavour
- Case study: $B \rightarrow D^* l \nu$
- Projects underway: more B semileptonic decay form factors, B mixing matrix elements, …

back-up

NRQCD matching

$$\langle \mathcal{J}^i \rangle = (1 + \alpha_s(\eta - \tau))\langle J^{(0)i}_{\rm latt} \rangle + \langle J^{(1)i}_{\rm latt} \rangle + e_4 \frac{\Lambda^2_{\rm QCD}}{m_b^2}$$

$$J^{(0)i}_{\rm latt}(x) = \bar{c}\gamma^i\gamma^5 Q$$
$$J^{(1)i}_{\rm latt}(x) = -\frac{1}{2am_b}\bar{c}\gamma^i\gamma^5\gamma\cdot\Delta Q$$

1-loop coefficients η & τ from Monahan, Shigemitsu, Horgan, PRD87 (2013)

Truncation errors enter at order: $\frac{\Lambda^2_{\rm QCD}}{m_b^2}$ included as Gaussian noise

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$\alpha_s \tau \langle J^{(0)}_{\mathrm{latt}} \rangle$	$\langle J^{(1)}_{\mathrm{latt}} \rangle$		
$0.00559(8)$	$0.0078(66)$	very coarse	& physical sea quark masses
$0.0064(1)$	$0.0055(48)$	coarse	
$0.0080(9)$	$0.0048(6)$	fine	

Cancellation expected from Luke's theorem

Chiral-continuum fit

Fit function:

Static limit

δ_a: disc errors

light quark mass

$$h_{A_1}(1) = (1+\delta_a^B)B + \delta_a^g \frac{g^2}{48\pi^2 f^2} \times \text{chiral logs} + C\frac{M_\pi^2}{\Lambda_\chi^2}$$

$$+ e_1\alpha_s^2\Big[1 + e_5(am_b-2)/2 + e_6((am_b-2)/2)^2\Big]J_{\text{latt}}^{(0)}$$

2-loop matching error

with g^2 = 0.53(8)

The α_s^2 uncertainty is the largest, by a factor of 2, compared to others